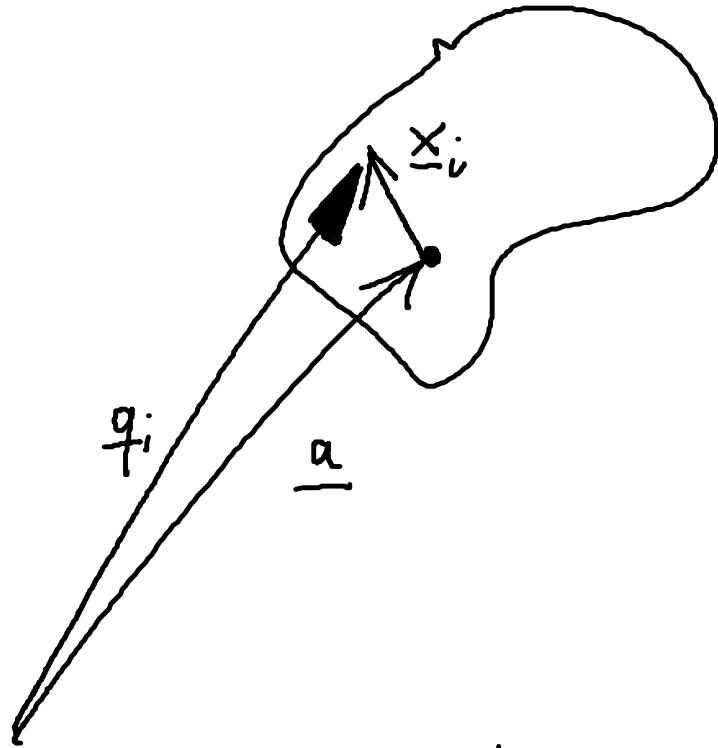


5.11.08



$$\underline{\ddot{a}} = \dot{0}$$

5.11.08

Drehimpuls

$$\begin{aligned} \underline{L} &= \sum_{i=1}^N m_i \underline{r}_i \times \dot{\underline{r}}_i \\ \underline{\dot{L}} &= \sum_{i=1}^N m_i \underline{r}_i \times \ddot{\underline{r}}_i = \\ &= \sum_{i=1}^N m_i \underline{a} \times \ddot{\underline{r}}_i + \sum_{i=1}^N m_i \underline{x}_i \times \left(\underset{=0}{\ddot{\underline{a}}} + \ddot{\underline{x}}_i \right) \\ &= \sum m_i \underline{a} \times \ddot{\underline{r}}_i + \underline{\dot{L}}_{\text{rot}} \end{aligned}$$

$$\dot{\underline{L}}_{\text{rot}} \equiv \sum_{i=1}^N m_i \underline{x}_i \times \ddot{\underline{x}}_i$$

externe Kraft auf MP i

Drehmoment $\underline{M} = \sum_{i=1}^N \underline{q}_i \times \underline{F}_i = \underline{\dot{L}}$

$$\begin{aligned} \underline{\dot{L}} &= \sum m_i \underline{a} \times \underline{q}_i + \underline{\dot{L}}_{\text{rot}} = \\ &= \sum_{i=1}^N \underline{q}_i \times \underline{F}_i^{\text{ext}} = \sum_{i=1}^N \underline{a} \times \underline{F}_i^{\text{ext}} + \sum_{i=1}^N \underline{x}_i \times \underline{F}_i^{\text{ext}} \end{aligned}$$

gesamte Kraft $\underline{F}_{\text{ges}}^{\text{ext}} = \sum_{i=1}^N \underline{F}_i^{\text{ext}}$

$$\underline{\dot{L}}_{\text{rot}} = \sum_{i=1}^N \underline{x}_i \times \underline{F}_i^{\text{ext}} \equiv \underline{M}_{\text{rot}}$$

Jetzt ins K' -System transformieren

$$\underline{M}'_{\text{rot}} = \underline{M}_{\text{rot}} = \underline{\dot{L}}_{\text{rot}} = R \left(\underline{\dot{L}}'_{\text{rot}} + \underline{\omega}' \times \underline{L}'_{\text{rot}} \right)$$

durch die Rotation

$$\Rightarrow \underline{M}'_{\text{rot}} = \underline{\dot{L}}'_{\text{rot}} + \underline{\omega}' \times \underline{L}'_{\text{rot}}$$

in K' -System

Führt $\underline{L}'_{rot} = \underline{\underline{\theta}}' \underline{\underline{\omega}}'$, $\underline{\underline{\theta}}'$ ist zeitlich konstant in K'

$$\underline{L}'_{rot} = \underline{\underline{\theta}}' \underline{\dot{\omega}}'$$

einsetzen:

$$\underline{M}'_{rot} = \underline{\underline{\theta}}' \underline{\dot{\omega}}' + \underline{\omega}' \times (\underline{\underline{\theta}}' \underline{\omega}')$$

$\underline{\omega}' = \underline{\omega}'(t)$ ist die gesuchte Größe

1. Fall: Der kräftefreie Kreis $\underline{M}'_{rot} = 0$

Im HA System gilt

$$\underline{\underline{\theta}}' \underline{\omega}' = \begin{pmatrix} \theta_1' \omega_1' \\ \theta_2' \omega_2' \\ \theta_3' \omega_3' \end{pmatrix}$$

3. Komponente der Eulerschen Gleichung

$$-\theta_3' \dot{\omega}_3' = \begin{vmatrix} \overset{+}{\omega_1'} & \overset{-}{\omega_2'} & \overset{+}{\omega_3'} \\ \theta_1' \omega_1' & \theta_2' \omega_2' & \theta_3' \omega_3' \end{vmatrix} = \omega_1' \omega_2' (\theta_2' - \theta_1') + \text{zyklisch}$$

$$\theta_2' \dot{\omega}_2' = \omega_3' \omega_1' (\theta_3' - \theta_1')$$

$$\theta_1' \dot{\omega}_1' = \omega_2' \omega_3' (\theta_2' - \theta_3')$$

Symmetrischer Kreisler : $\theta_1' = \theta_2' > \theta_3'$
 Kugelkreisler : $\theta_1' = \theta_2' = \theta_3'$

$$-\theta_3' \dot{\omega}_3' = 0 \Rightarrow \omega_3' = \text{const} !$$

$$\omega_1' \dot{\omega}_1' = \omega_2' \omega_3' (\theta_1' - \theta_3')$$

$$\omega_2' \dot{\omega}_2' = \omega_1' \omega_3' (\theta_3' - \theta_1') \quad / \cdot i$$

$$\omega_{\perp}(t) = \omega_1(t) + i \omega_2(t)$$

$$\omega_1' (\dot{\omega}_1' + i \dot{\omega}_2') = \omega_3' (\theta_3' - \theta_1') i (\omega_1' + i \omega_2')$$

$$\Rightarrow \dot{\omega}_{\perp}(t) = \underbrace{\omega_3' \frac{\theta_3' - \theta_1'}{\theta_1'}}_{\equiv \omega_0'} i \omega_{\perp}(t)$$

$$\Rightarrow \text{vom Typ} \quad \dot{f}(t) = \alpha f(t)$$

$$\text{Lösung} \quad \omega_{\perp}(t) = \omega_{\perp}' e^{i \omega_0' t}$$

jetzt zeichnen

$$\omega_1'(t) = \omega_{\perp}' \cos(\omega_0' t)$$

$$\omega_2'(t) = \omega_{\perp}' \sin(\omega_0' t)$$

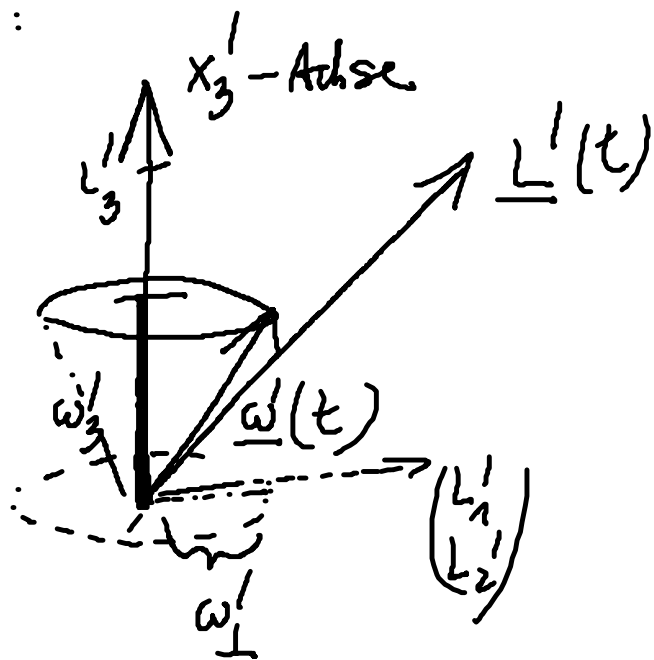
$$\text{(Euler: } e^{ix} = \cos x + i \sin x \text{)}$$

Der Betrag von $\underline{\omega}'(t)$ ist gegeben durch

$$|\underline{\omega}'(t)|^2 = \omega_{\perp}'^2 + (\omega_3')^2 = \text{const}$$

geom. Interpretation:

Im K' -System:



$$L_3' = \theta_3' \omega_3' = \text{const}$$

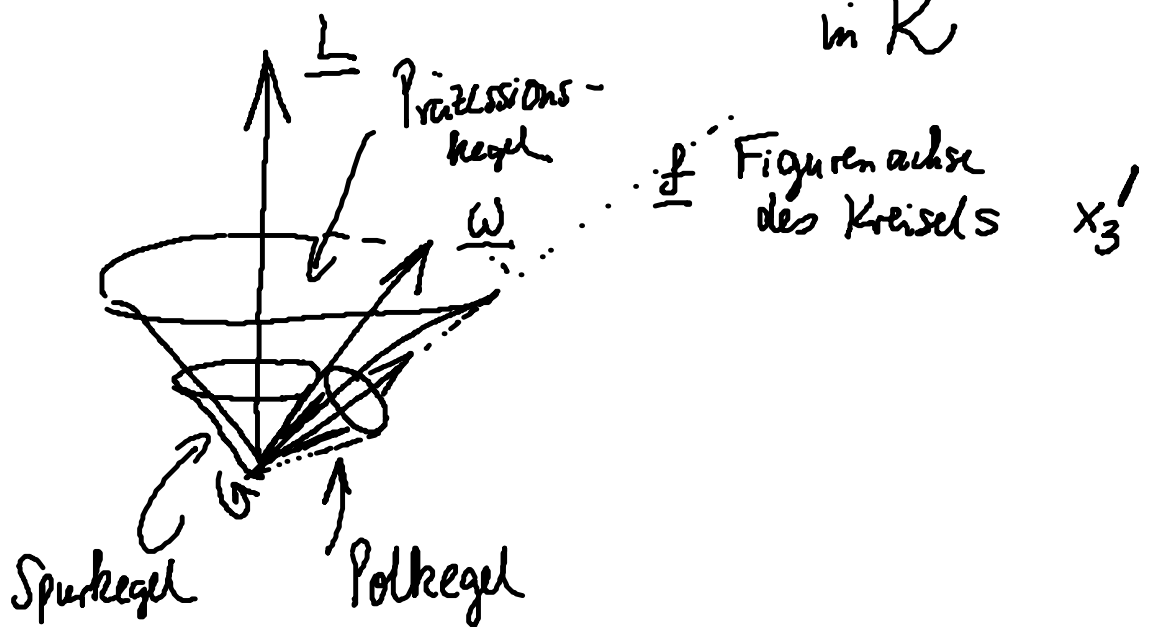
$$L_1'(t) = \theta_1' \omega_1'(t) = \theta_1' \omega_{\perp}' \cos(\omega_0' t)$$

$$L_2'(t) = \theta_1' \omega_2'(t) = \theta_1' \omega_{\perp}' \sin(\omega_0' t)$$

Es liegen also $\underline{e}'_3, \underline{\omega}', \underline{L}'$ in
 einer gemeinsamen Ebene, die um die
 \underline{e}'_3 -Achse rotiert mit
 der Winkelgeschwindigkeit ω_0' .

Wie sieht das in K aus?

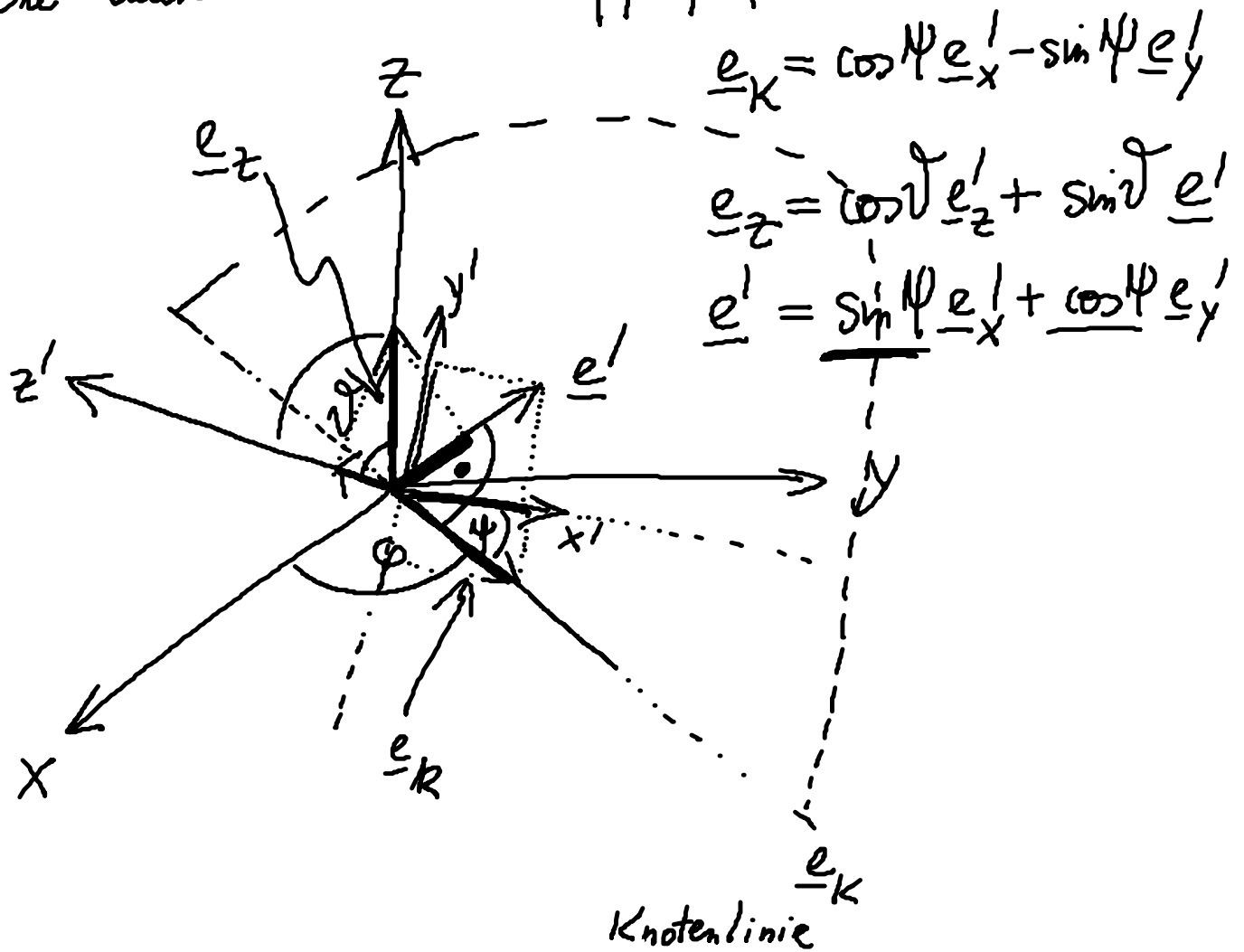
Dort ist $\underline{L} = \text{const!}$



„reguläre Präzession“

- Winkelgeschwindigkeit $\omega_0' = \omega_3' \frac{\theta_3' - \theta_1'}{\theta_1'}$

Drei Eulerschen Winkel φ, ϑ, ψ



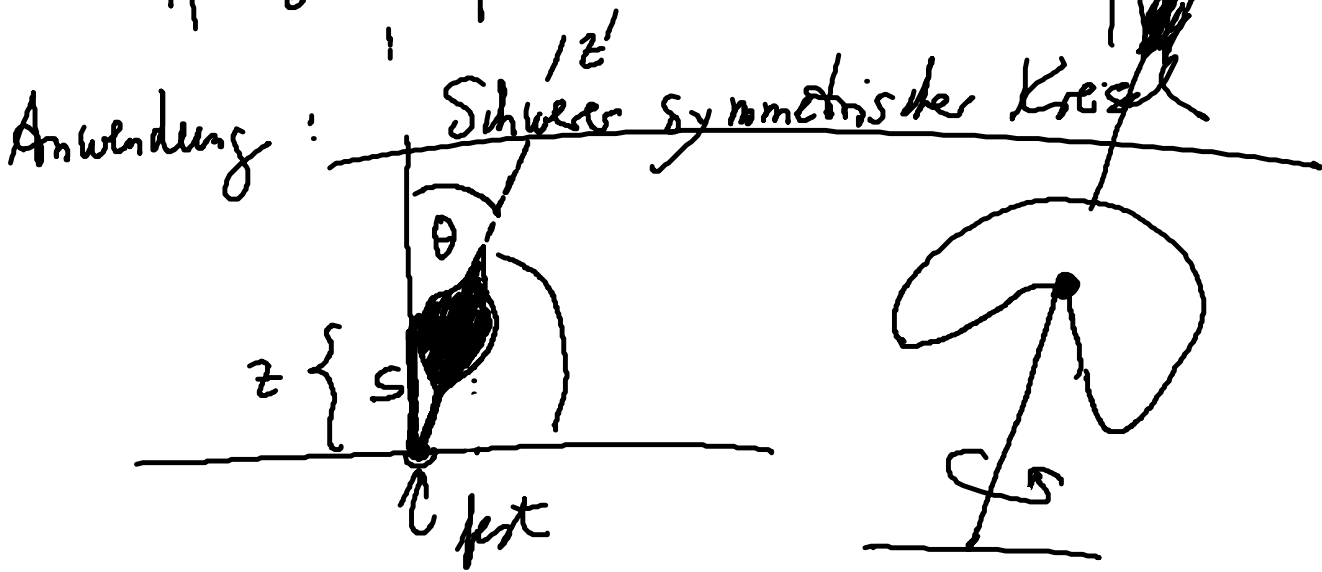
Damit

$$\underline{e}_z = \sin\psi \sin\vartheta \underline{e}_x' + \cos\psi \sin\vartheta \underline{e}_y' + \cos\vartheta \underline{e}_z'$$

Für den Vektor der Winkelgeschwindigkeit gilt

$$\begin{aligned} \underline{\omega} &= \dot{\vartheta} \underline{e}_k + \dot{\varphi} \underline{e}_z + \dot{\psi} \underline{e}_z' \\ &= \omega_x' \underline{e}_x' + \omega_y' \underline{e}_y' + \omega_z' \underline{e}_z' \end{aligned}$$

$$\Rightarrow \begin{cases} \omega'_x = \dot{\varphi} \sin \vartheta \sin \psi + \dot{\vartheta} \cos \psi \\ \omega'_y = \dot{\varphi} \sin \vartheta \cos \psi - \dot{\vartheta} \sin \psi \\ \omega'_z = \dot{\varphi} \cos \vartheta + \dot{\psi} \end{cases}$$



$$V = Mg z = Mg s \cos \vartheta \quad \text{potentielle Energie}$$

$$T_{\text{rot}} = \frac{1}{2} \underline{\omega}' \underline{\underline{\theta}}' \underline{\omega}' = \frac{1}{2} \left(\omega_x'^2 \theta_1' + \omega_y'^2 \theta_2' + \omega_z'^2 \theta_3' \right)$$

sei diagonal

Symm. Kreisel: $\theta_1' = \theta_2'$

Setzt Lagrange II mit den Koordinaten φ, ϑ, ψ

$$\Rightarrow L = T - V =$$

$$= \frac{1}{2} \left((\dot{\varphi}^2 \sin^2 \vartheta + \dot{\vartheta}^2) \theta_1' + (\dot{\varphi} \cos \vartheta + \dot{\psi})^2 \theta_3' \right)$$

$$- M g s \cos \vartheta$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0$$

L hängt nicht von φ und ψ ab
(nur von $\dot{\varphi}$, $\dot{\psi}$)

$$\Rightarrow \frac{\partial L}{\partial \varphi} = 0 \quad \varphi \text{ ist zyklisch}$$

$$\dot{L}_z \equiv \frac{d}{dt} \frac{\partial L}{\partial \dot{\varphi}} = 0 \Rightarrow L_z = \text{const}$$

$$\frac{\partial L}{\partial \psi} = 0 \Rightarrow \frac{d}{dt} \frac{\partial L}{\partial \dot{\psi}} \equiv \dot{L}'_z = 0$$

$$\Rightarrow L'_z = \frac{\partial L}{\partial \dot{\psi}} = \text{const}$$

Damit haben wir bereits
zwei Konstanten der Bewegung.