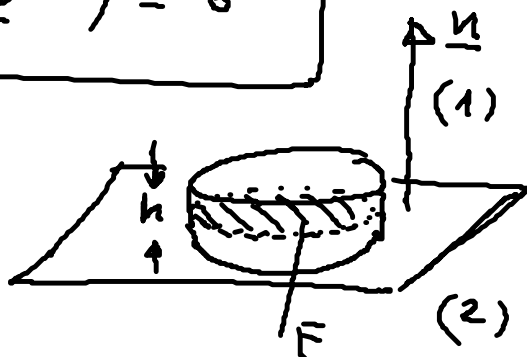


# Grenzbed. für Felder

Normalkomponenten:

$$\underline{n} \cdot (\underline{B}^{(1)} - \underline{B}^{(2)}) = 0$$

$$\underline{n} \cdot (\underline{D}^{(1)} - \underline{D}^{(2)}) = \sigma$$



## Tangentialkomp.

Verallg. Gauß'scher Satz:

$$(1) \int_V d^3r \nabla \times \underline{E} = \oint_{\partial V} d\underline{f} \times \underline{E} = - \int_V d^3r \dot{\underline{B}}$$

$$(2) \int_V d^3r \nabla \times \underline{H} = \oint_{\partial V} d\underline{f} \times \underline{H} = \int_V d^3r (\underline{j} + \dot{\underline{D}})$$

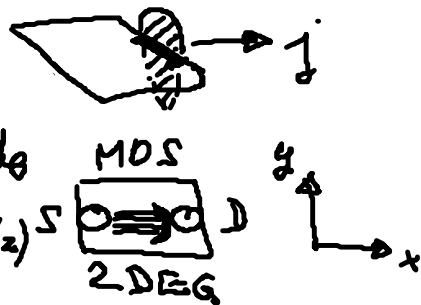
$h \rightarrow 0$  (1)  $\Rightarrow \lim_{h \rightarrow 0} \oint_{\partial V} d\underline{f} \times \underline{E} = \int_F d\underline{f} \underline{n} \times (\underline{E}^{(1)} - \underline{E}^{(2)})$   
 (Tangentialkomponente)

$$(2) \Rightarrow \lim_{h \rightarrow 0} \oint_{\partial V} d\underline{f} \times \underline{H} = \int_F d\underline{f} \underline{n} \times (\underline{H}^{(1)} - \underline{H}^{(2)})$$

Annahme: Grenzfläche trägt

freie Flächenstromdichte

$$\underline{j} : \underline{j}(\underline{r}, t) = \underline{j}(x, y, t) \delta(z) \delta^2$$



$$\Rightarrow \lim_{h \rightarrow 0} \int_V d^3r \underline{j} = \int_F d\underline{f} \underline{j}$$

$\underline{B}$ ,  $\underline{D}$  und  $\dot{\underline{B}}$ ,  $\dot{\underline{D}}$  sind beschränkt:

$$\Rightarrow \lim_{h \rightarrow 0} \int_V d^3r \dot{\underline{B}} = \lim_{h \rightarrow 0} \int_V d^3r \dot{\underline{D}} = 0$$

Also für bel. Fläche  $F$ :

$$\int_F d\vec{f} \underbrace{\underline{n} \times (\underline{E}^{(1)} - \underline{E}^{(2)})}_0 = 0$$

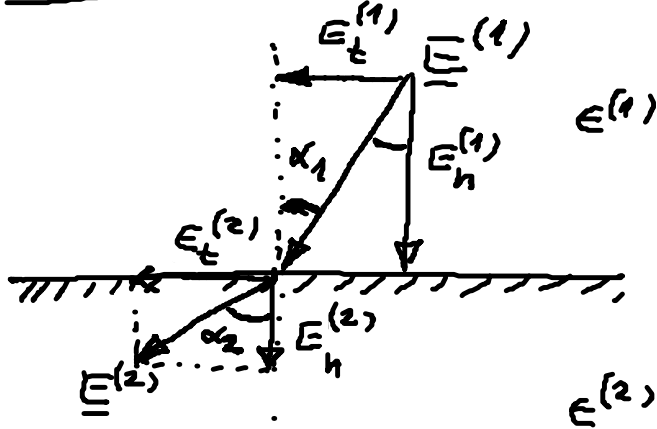
$$\int_F d\vec{f} \underbrace{\underline{n} \times (\underline{H}^{(1)} - \underline{H}^{(2)})}_= \int_F d\vec{f} \underbrace{g}_{\underline{K}}$$

Zus. fassung:  $\delta \underline{E} := \underline{E}^{(1)} - \underline{E}^{(2)}$  usw.

Maxwell-Gln.	Grenzbed.	
$\nabla \times \underline{E} = -\dot{\underline{B}}$	$\underline{n} \times \delta \underline{E} = 0$	Tang. Komp. v. $\underline{E}$ stetig
$\nabla \cdot \underline{D} = \rho$	$\underline{n} \cdot \delta \underline{D} = \sigma$	Normalkomp. v. $\underline{D}$ springt (Flächendis.)
$\nabla \times \underline{H} = \underline{j} + \dot{\underline{D}}$	$\underline{n} \times \delta \underline{H} = \underline{g}$	Tang. Komp. v. $\underline{H}$ springt (Flächenrot.)
$\nabla \cdot \underline{B} = 0$	$\underline{n} \cdot \delta \underline{B} = 0$	Normalkomp. v. $\underline{B}$ stetig

Beispiel:

(1) Grenzfläche zwischen 2 dielektr. Materialien  
mit  $\epsilon^{(1)} < \epsilon^{(2)}$  und  $\sigma = 0$ :  $\frac{\epsilon^{(1)}}{\epsilon^{(2)}}$



$$\underline{E}^{(i)} = \underline{E}_t^{(i)} + \underline{E}_n^{(i)}$$

mit  $\underline{E}_t^{(1)} = \underline{E}_t^{(2)}$   
 $\underline{D}_n^{(1)} = \underline{D}_n^{(2)}$   
 $\Rightarrow \epsilon_1 \underline{E}_n^{(1)} = \epsilon_2 \underline{E}_n^{(2)}$   
 $\Rightarrow \underline{E}_n^{(2)} = \frac{\epsilon_1}{\epsilon_2} \underline{E}_n^{(1)}$

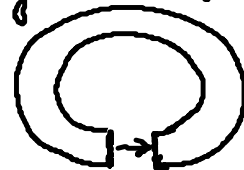
$$\tan \alpha_1 = \frac{|E_t^{(1)}|}{|E_n^{(1)}|} = \frac{\epsilon_1 |E_t^{(2)}|}{\epsilon_2 |E_n^{(2)}|} = \frac{\epsilon_1}{\epsilon_2} \tan \alpha_2$$

(Brechungsgesetz des Feldlinien)

(2) Grenzfläche zwischen Vakuum (Luft) und magn. Material

(2.1) Speziell  $\underline{B} \perp$  Grenzfläche (z.B. zwischen den Polschalen eines Ringmagneten)

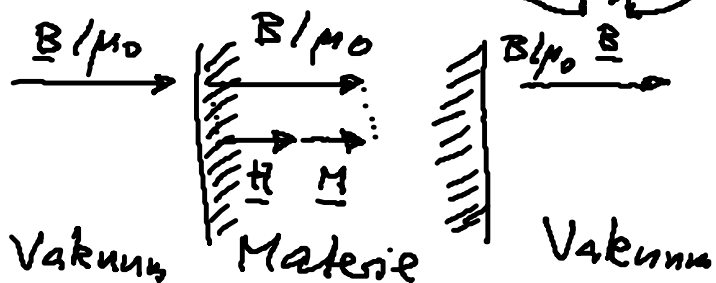
$\Rightarrow \underline{B}$  stetig



(a) Paramagnetisch

$$\frac{1}{\mu_0} \underline{B} = \underline{H} + \underline{M}$$

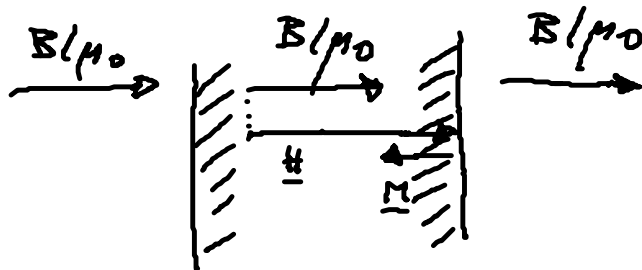
$\underline{H} \uparrow \uparrow \underline{M}$



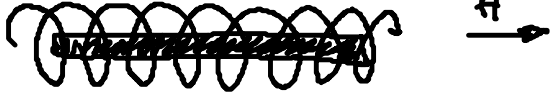
(b) Diamagnetisch

$$\frac{1}{\mu_0} \underline{B} = \underline{H} + \underline{M}$$

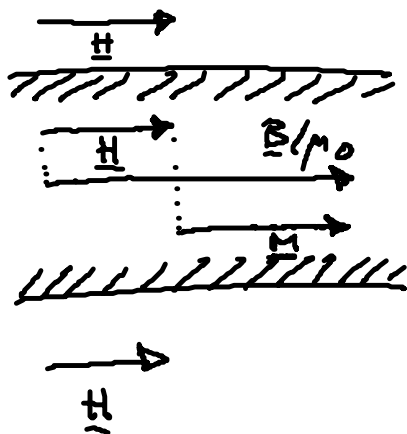
$\underline{H} \uparrow \downarrow \underline{M}$



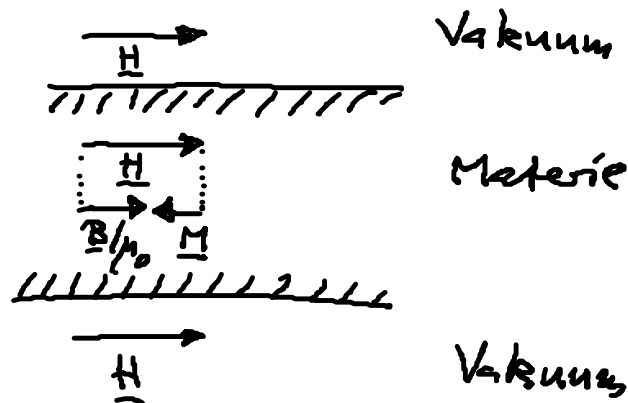
(2.2) Sei  $\underline{B} \parallel$  Grenzfläche (z.B. lange Spule)

$\Rightarrow \underline{H}$  stetig (für  $\underline{g} = 0$ ) 

(a) paramagnetisch



(b) diamagnetisch



## 5.5 Mikroskop. klass. Modell der Polarisierbarkeit

Ziel: berechne  $\chi_e$

Methode: berechne induzierte mittl. el. Dipoldichte  $\underline{P}$   
für geg. Feld  $\underline{E}$   
(induzierte Polarisation)

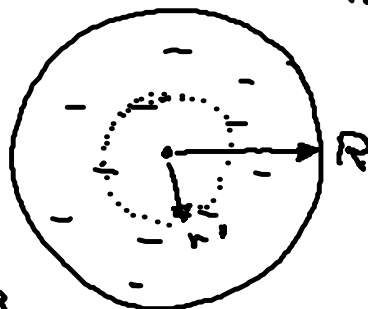
klass. Atommodell: homog. geladene Kugel mit Radius  $R$   
u. el. Ladung  $Q_e = -Ze < 0$   
pktförmiger Kern mit  $Q_k = Ze > 0$

el. Feld der el.  $\underline{E}_e(\underline{r})$ : bei  $\underline{r}_k$

Gauß:

$$\epsilon_0 \oint_{\partial V} \underline{E}_e(\underline{r}) \cdot d\underline{r} = \int_V \rho_e(\underline{r}') d^3r'$$

$$= \int_V d^3r' \frac{Q_e}{\frac{4\pi}{3} R^3}$$



$$4\pi r^2 \epsilon_0 |\underline{E}_e(r)| = Q_e \frac{r^{-2}}{R^3} \quad \begin{array}{l} r_e \text{ Zentrum der El. Ladung} \\ \underline{r}' = \underline{r} - \underline{r}_e \end{array}$$

$$\underline{E}_e(r) = \frac{Q_e}{4\pi\epsilon_0} \frac{\underline{r} - \underline{r}_e}{R^3}$$

Kraft auf den Kern:

$$\begin{aligned} \underline{F}_K &= Q_K \underline{E}_e(\underline{r}_K) = \frac{Q_K Q_e}{4\pi\epsilon_0 R^3} (\underline{r}_K - \underline{r}_e) \\ &= -\frac{Z^2 e^2}{4\pi\epsilon_0 R^3} (\underline{r}_K - \underline{r}_e) \end{aligned}$$

Kraft des Kerns auf El.:

$$\underline{F}_e = -\underline{F}_K$$

Bew. gl. (mit äußerem el. Feld  $\underline{E}_a$ ):

$$m_K \ddot{\underline{r}}_K = \underline{F}_K + Q_K \underline{E}_a = -\frac{Z^2 e^2}{4\pi\epsilon_0 R^3} (\underline{r}_K - \underline{r}_e) + Z e \underline{E}_a$$

$$Z m_e \ddot{\underline{r}}_e = \underline{F}_e + Q_e \underline{E}_a = \frac{Z^2 e^2}{4\pi\epsilon_0 R^3} (\underline{r}_K - \underline{r}_e) - Z e \underline{E}_a$$

$$\Rightarrow \underbrace{\ddot{\underline{r}}_K - \ddot{\underline{r}}_e}_{\ddot{\underline{r}} \text{ (Rel. Koord.)}} = -\frac{Z^2 e^2}{4\pi\epsilon_0 R^3} \underbrace{(\underline{r}_K - \underline{r}_e)}_{\underline{r}} + Z e \underbrace{\left(\frac{1}{m_K} + \frac{1}{Z m_e}\right)}_{\approx \frac{1}{Z m_e}} \underline{E}_a$$

$$\Rightarrow \boxed{\ddot{\underline{r}} + \omega_0^2 \underline{r} = \frac{e}{m_e} \underline{E}_a} \quad \text{mit } \omega_0^2 = \frac{Z e^2}{4\pi\epsilon_0 m_e R^3}$$

harmon. Osz. mit stat. Zustand  $\underline{r} = \frac{e}{m_e \omega_0^2} \underline{E}_a$

$\Rightarrow$  stat. mikr. el. Dipolmoment:

$$\underline{p} = \underbrace{\int_V d^3r' \rho_e(\underline{r}') \underline{r}'}_{\text{Kern}} + Z e \underbrace{\int_V d^3r' \delta(\underline{r}' - \underline{r}) \underline{r}'}_{\text{Elektron}}$$

$$-\frac{1}{4\pi\epsilon_0} \int d^3r' \frac{\rho(\mathbf{r}')}{r'^2} \quad 17$$

(Symm.)

$$\underline{p} = Ze\underline{r} = \frac{Ze^2}{m_e\omega_0^2} \underline{E}_a = \epsilon_0 \alpha \underline{E}_a$$

mit Polarisierbarkeit des Atoms

$$\alpha := \frac{Ze^2}{\epsilon_0 m_e \omega_0^2} = 4\pi R^3 = 3V_{\text{atom}}$$

- QM : induz. Dipol : zeitunabh. Stör. rechn. 2. Ordn.  
 $\rightarrow$  quadrat. Starkeffekt  
 perm. Dipol : zeitunabh. Stör. rechn. 1. Ordn.  
 $\rightarrow$  lin. Starkeffekt  $d_{13} = -3ea_0$

makroskop. gemittelte Dipoldichte :

$$\underline{P} = n \underline{p} = \epsilon_0 n \alpha \underline{E}_a \quad (n \text{ mittlere Atombichte})$$