

Grenzbed. für Felder

Normalkomponenten:

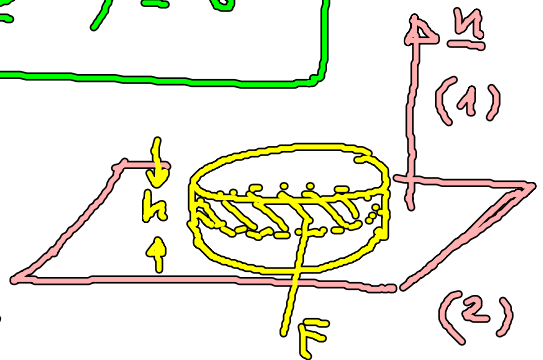
$$\begin{aligned} \underline{n} \cdot (\underline{E}^{(1)} - \underline{E}^{(2)}) &= 0 \\ \underline{n} \cdot (\underline{D}^{(1)} - \underline{D}^{(2)}) &= \sigma \end{aligned}$$

Tangentialkomp.

Verallg. Gauß'scher Satz:

$$(1) \int_V d^3r \nabla \times \underline{E} = \int_{\partial V} d\underline{f} \times \underline{E} = - \int_V d^3r \dot{\underline{B}}$$

$$(2) \int_V d^3r \nabla \times \underline{H} = \int_{\partial V} d\underline{f} \times \underline{H} = \int_V d^3r (\underline{j} + \dot{\underline{D}})$$

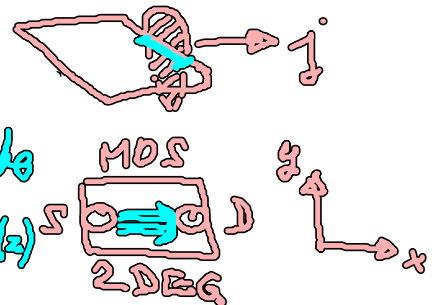


$\underline{h} \rightarrow 0$ (1) $\Rightarrow \lim_{h \rightarrow 0} \int_{\partial V} d\underline{f} \times \underline{E} = \int_F d\underline{f} \underline{n} \times (\underline{E}^{(1)} - \underline{E}^{(2)})$
(Tangentialkomponente)

(2) $\Rightarrow \lim_{h \rightarrow 0} \int_{\partial V} d\underline{f} \times \underline{H} = \int_F d\underline{f} \underline{n} \times (\underline{H}^{(1)} - \underline{H}^{(2)})$

Annahme: Grenzfläche trägt freie Flächenstromdichte

\underline{j} : $\underline{j}(\underline{r}, t) = g(x, y, t) \underline{S}(\underline{r})$



$\Rightarrow \lim_{h \rightarrow 0} \int_V d^3r \underline{j} = \int_F d\underline{f} \underline{g}$

\underline{B} , \underline{D} und $\dot{\underline{B}}$, $\dot{\underline{D}}$ sind beschränkt:

$$\Rightarrow \lim_{h \rightarrow 0} \int_V d^3r \underline{\dot{B}} = \lim_{h \rightarrow 0} \int_V d^3r \underline{\dot{D}} = 0$$

Also für bel. Fläche F :

$$\int_F d\vec{f} \underline{n} \times (\underline{E}^{(1)} - \underline{E}^{(2)}) = 0$$

$$\int_F d\vec{f} \underline{n} \times (\underline{H}^{(1)} - \underline{H}^{(2)}) = \int_F d\vec{f} \underline{q}$$

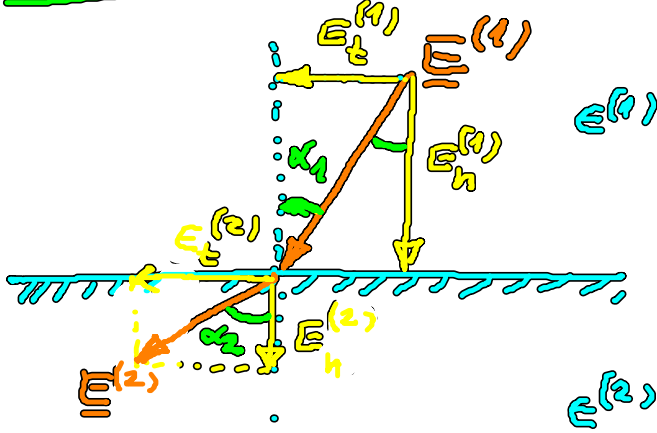
Zus. fassung: $\delta \underline{E} := \underline{E}^{(1)} - \underline{E}^{(2)}$ usw.

Maxwell-Gln.	Grenzbed.
$\nabla \times \underline{E} = -\underline{\dot{B}}$	$\underline{n} \times \delta \underline{E} = 0$
$\nabla \cdot \underline{D} = \rho$	$\underline{n} \cdot \delta \underline{D} = \sigma$
$\nabla \times \underline{H} = \underline{j} + \underline{\dot{D}}$	$\underline{n} \times \delta \underline{H} = \underline{q}$
$\nabla \cdot \underline{B} = 0$	$\underline{n} \cdot \delta \underline{B} = 0$

Tang. Komp. v. \underline{E} stetig
 Normalkomp. v. \underline{D} springt
 (Flächendiv.)
 Tang. Komp. v. \underline{H} springt
 (Flächenrot.)
 Normalkomp. v. \underline{B} stetig

Beispiel:

(1) Grenzfläche zwischen 2 dielektr. Materialien
 mit $\epsilon^{(1)} < \epsilon^{(2)}$ und $\sigma = 0$: $\frac{\epsilon^{(1)}}{\epsilon^{(2)}}$



$$\begin{aligned} \underline{E}^{(1)} &= \underline{E}_t^{(1)} + \underline{E}_n^{(1)} \\ \text{mit } \underline{E}_t^{(1)} &= \underline{E}_t^{(2)} \\ \underline{D}_n^{(1)} &= \underline{D}_n^{(2)} \\ \Rightarrow \epsilon_1 \underline{E}_n^{(1)} &= \epsilon_2 \underline{E}_n^{(2)} \\ \Rightarrow \underline{E}_n^{(2)} &= \frac{\epsilon_1}{\epsilon_2} \underline{E}_n^{(1)} \end{aligned}$$

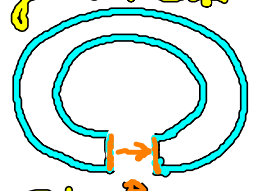
$$\tan \alpha_1 = \frac{|E_t^{(1)}|}{|E_n^{(1)}|} = \frac{\epsilon_1 |E_t^{(2)}|}{\epsilon_2 |E_n^{(2)}|} = \frac{\epsilon_1}{\epsilon_2} \tan \alpha_2$$

(Brechungsgesetz der Feldlinien)

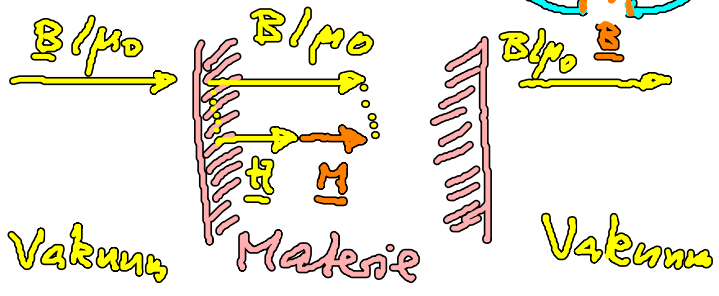
(2) Grenzfläche zwischen Vakuum (Luft) und magn. Material

(2.1) Speziell $\underline{B} \perp$ Grenzfläche (z.B. zwischen den Polschüben eines Ringmagneten)

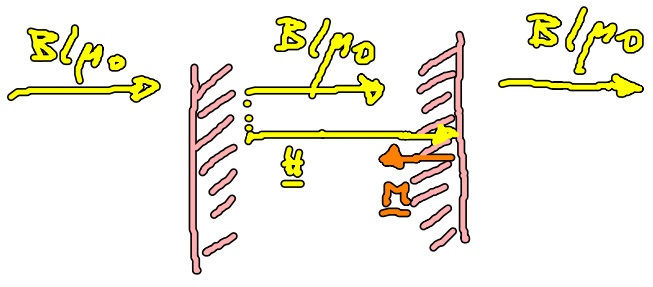
$\Rightarrow \underline{B}$ stetig



(a) Paramagnetisch
 $\frac{1}{\mu_0} \underline{B} = \underline{H} + \underline{M}$
 $\underline{H} \uparrow \uparrow \underline{M}$

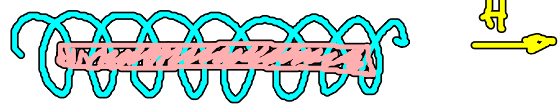


(b) Diamagnetisch
 $\frac{1}{\mu_0} \underline{B} = \underline{H} + \underline{M}$
 $\underline{H} \uparrow \downarrow \underline{M}$

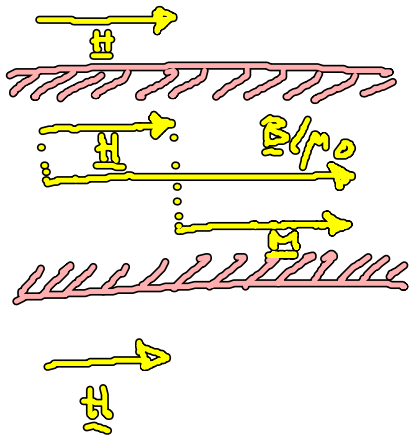


(2.2) Sei $\underline{B} \parallel$ Grenzfläche (z.B. lange Spule)

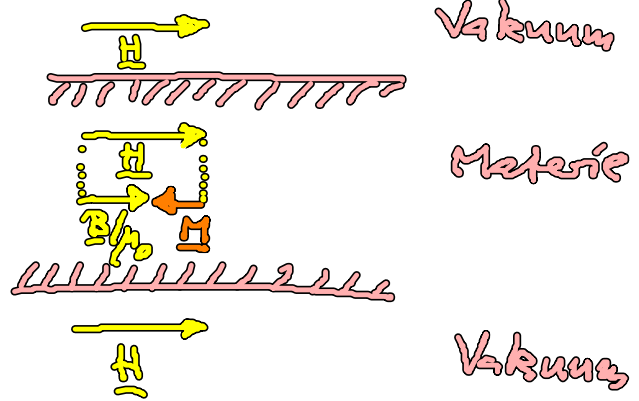
$\Rightarrow \underline{H}$ stetig (für $g=0$)



(a) paramagnetisch



(b) diamagnetisch



5.5 Mikrokop. klass. Modell der Polarisierbarkeit

Ziel: berechne χ_e

Methode: berechne induzierte mittl. el. Dipoldichte \underline{P}
für geg. Feld \underline{E}
(induzierte Polarisation)

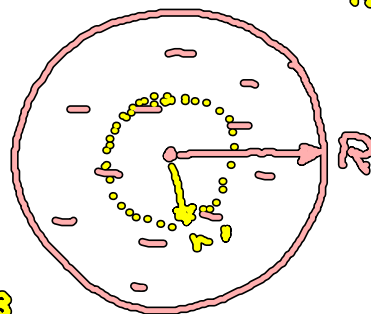
klass. Atommodell: homog. geladene Kugel mit Radius R
u. el. Ladung $Q_e = -Ze < 0$
pktförmiger Kern mit $Q_k = Ze > 0$

el. Feld der el. $\underline{E}_e(x)$: bei r_k

Gauß:

$$\epsilon_0 \oint_{\partial V} \underline{E}_e(x) \cdot d\underline{r} = \int_V \rho_e(r) dV$$

$$= \int_V \rho_e(r) dV = \frac{Q_e}{\frac{4\pi}{3} R^3}$$



$$4\pi r^2 \epsilon_0 |\underline{E}_e(r)| = Q_e \frac{r^2}{R^3} \quad \Gamma_e \text{ Zentrum der El. Ladung}$$

$$\underline{r}' = \underline{r} - \underline{r}_e$$

$$\underline{E}_e(r) = \frac{Q_e}{4\pi\epsilon_0} \frac{\underline{r} - \underline{r}_e}{R^3}$$

Kraft auf den Kern:

$$\underline{F}_k = Q_k \underline{E}_e(\underline{r}_k) = \frac{Q_k Q_e}{4\pi\epsilon_0 R^3} (\underline{r}_k - \underline{r}_e)$$

$$\text{Kraft des Kerns auf El.:} = -\frac{Z^2 e^2}{4\pi\epsilon_0 R^3} (\underline{r}_k - \underline{r}_e)$$

$$\underline{F}_e = -\underline{F}_k$$

Bew.gl. (mit äußerem el. Feld \underline{E}_a):

$$m_k \ddot{\underline{r}}_k = \underline{F}_k + Q_k \underline{E}_a = -\frac{Z^2 e^2}{4\pi\epsilon_0 R^3} (\underline{r}_k - \underline{r}_e) + Z e \underline{E}_a$$

$$Z m_e \ddot{\underline{r}}_e = \underline{F}_e + Q_e \underline{E}_a = \frac{Z^2 e^2}{4\pi\epsilon_0 R^3} (\underline{r}_k - \underline{r}_e) - Z e \underline{E}_a$$

$$\Rightarrow \underbrace{\ddot{\underline{r}}_k - \ddot{\underline{r}}_e}_{\ddot{\underline{r}} \text{ (Rel. bew.)}} = -\frac{Z^2 e^2}{4\pi\epsilon_0 R^3} \underbrace{\left(\frac{1}{m_k} + \frac{1}{Z m_e} \right)}_{\approx \frac{1}{Z m_e}} (\underline{r}_k - \underline{r}_e) + Z e \underbrace{\left(\frac{1}{m_k} + \frac{1}{Z m_e} \right)}_{\approx \frac{1}{Z m_e}} \underline{E}_a$$

$$\Rightarrow \boxed{\ddot{\underline{r}} + \omega_0^2 \underline{r} = \frac{e}{m_e} \underline{E}_a} \quad \text{mit } \omega_0^2 = \frac{Z e^2}{4\pi\epsilon_0 m_e R^3}$$

harmon. Osz. mit stat. Zustand $\underline{r} = \frac{e}{m_e \omega_0^2} \underline{E}_a$

\Rightarrow stat. mikro. el. Dipolmoment:

$$\underline{p} = \underbrace{\int_V d^3r' \rho_e(\underline{r}') \underline{r}'} + Z e \underbrace{\int_V d^3r' \delta(\underline{r}' - \underline{r}) \underline{r}'}$$

$$-\frac{1}{4\pi\epsilon_0} \int d^3r' \frac{\rho(\mathbf{r}')}{r^2} \quad \text{in}$$

$$\underbrace{\quad}_{0 \text{ (Symm.)}}$$

$$\underline{p} = Ze\underline{r} = \frac{Ze^2}{m_e\omega_0^2} \underline{E}_a = \epsilon_0 \alpha \underline{E}_a$$

mit Polarisierbarkeit des Atoms

$$\alpha := \frac{Ze^2}{\epsilon_0 m_e \omega_0^2} = 4\pi R^3 = 3V_{\text{atom}}$$

QM : induz. Dipol : zeitunabh. Stör. rechn. 2. Ord.
 \rightarrow quadrat. Starkeffekt
 perm. Dipol : zeitunabh. Stör. rechn. 1. Ord.
 \rightarrow lin. Starkeffekt $d_{13} = -3ea_0$

makroscop. gemittelte Dipoldichte :

$$\underline{P} = n \underline{p} = \epsilon_0 n \alpha \underline{E}_a \quad (n \text{ mittlere Atombichte})$$