C) Kohärenter Laserstrahl

Eigenwertglg von c: $\quad c|\alpha\rangle = \alpha|\alpha\rangle$

α: Eigenwert von c, komplex $\quad \alpha = |\alpha| \exp\{i\varphi\}$

$$\alpha = \langle\alpha|c|\alpha\rangle = \langle c^+|\alpha\rangle|\alpha\rangle = \langle\alpha|c^+|\alpha\rangle^*$$

$$\Rightarrow \quad c^+|\alpha\rangle = \alpha^*|\alpha\rangle$$

$$\hat{\vec{E}} = i\sqrt{\frac{\hbar\omega}{2\varepsilon V}}\,\vec{u}\left[\exp\{i(\vec{q}\cdot\vec{r}-\omega t)\}c - \exp\{-i(\vec{q}\cdot\vec{r}-\omega t)\}c^+\right]$$

$$\langle\alpha|\vec{E}|\alpha\rangle = i\sqrt{\frac{\hbar\omega}{2\varepsilon V}}\,\vec{u}\left[\exp\{i(\vec{q}\cdot\vec{r}-\omega t)\}\alpha - \exp\{-i(\vec{q}\cdot\vec{r}-\omega t)\}\alpha^*\right]$$

$$= i\sqrt{\frac{\hbar\omega}{2\varepsilon V}}\,|\alpha|\,\vec{u}\left[\exp\{i(\vec{q}\cdot\vec{r}-\omega t+\varphi)\} - \exp\{-i(\vec{q}\cdot\vec{r}-\omega t+\varphi)\}\right]$$

$$= -\sqrt{\frac{2\hbar\omega}{\varepsilon V}}\,|\alpha|\,\vec{u}\,\sin\{\vec{q}\cdot\vec{r}-\omega t+\varphi\}$$

$$[c, c^+] = 1 \quad \text{oder} \quad c c^+ = c^+ c + 1$$

$$[c, c^{+n}] = c\,c^{+n} - c^{+n}c$$
$$= c^+ c\, c^{+(n-1)} + c^{+(n-1)} - c^{+n}c$$
$$= c^{+2} c\, c^{+(n-2)} + c^{+(n-1)} + c^{+(n-1)} - c^{+n}c$$
$$= c^{+n}c + n\,c^{+(n-1)} - c^{+n}c = n\,c^{+(n-1)}$$

<u>Beweis</u> von $\exp\{a+b\} = \exp\{a\}\exp\{b\}\exp\{-\frac{x}{2}\}$

mit $[a,b]=x$, $[a,x]=0=[b,x]$, $[a,b^n]=n\,b^{n-1}x$; $[a^n,b]=n\,a^{n-1}x$

1) $\exp\{a+b\} = \sum_{n=0}^{\infty} \frac{1}{n!}(a+b)^n$

$= 1+(a+b)+\frac{1}{2}(a+b)^2+\frac{1}{3!}(a+b)^3+\frac{1}{4!}(a+b)^4+\frac{1}{5!}(a+b)^5$

2) $\exp\{a\}\exp\{b\} = \left(1+a+\frac{1}{2}a^2+\frac{1}{3!}a^3+\ldots\right)\left(1+b+\frac{1}{2}b^2+\frac{1}{3!}b^3+\ldots\right)$

$(a+b)_g^2 = a^2+2ab+b^2$

$(a+b)_g^3 = a^3+3a^2b+3ab^2+b^3$

$(a+b)_g^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$

$(a+b)_g^5 = a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$

3) $(a+b)^2 = a^2+ab+ba+b^2 = (a+b)_g^2 - x$

$(a+b)^3 = a^3+a^2b+aba+ab^2+ba^2+bab+b^2a+b^3$

$= (a+b)_g^3 - 3x(a+b)$

$(a+b)^4 = (a+b)_g^4 - 6x(a+b)_g^2 + 3x^2$

$(a+b)^5 \stackrel{!}{=} (a+b)_g^5 - 10(a+b)_g^3 x + 15(a+b)x^2$

4) $\exp\{a+b\} = 1+(a+b)+\frac{1}{2}(a+b)^2+\frac{1}{3!}(a+b)^3+\ldots$

$= \exp\{a\}\exp\{b\}\exp\{-\frac{x}{2}\}$