5.2 Feldoperatoren   Bosonen

$$[\hat{\psi}(\underline{x}), \hat{\psi}^{+}(\underline{x}')] =$$

$$= \left[\sum_{\nu=1}^{\infty} \psi_\nu(\underline{x})\, a_\nu , \sum_{\mu=1}^{\infty} \psi_\mu^*(\underline{x}')\, a_\mu^+ \right] = \sum_{\nu,\mu}^{1\ldots\infty} \psi_\nu(\underline{x})\, \psi_\mu^*(\underline{x}') \left[a_\nu, a_\mu^+ \right]$$

$$= \sum_{\nu,\mu}^{1\ldots\infty} \psi_\nu(\underline{x})\, \psi_\mu^*(\underline{x}')\, \delta_{\nu\mu}\, 1 = \sum_{\nu=1}^{\infty} \psi_\nu(\underline{x})\, \psi_\nu^*(\underline{x}')\, 1 = \delta(\underline{x} - \underline{x}')\, 1$$

$$\hat{N} = \sum_{\lambda=1}^{\infty} a_\lambda^+ a_\lambda = \sum_{\lambda=1}^{\infty} \int \psi_\lambda^*(\underline{x})\, \hat{\psi}(\underline{x})\, d\tau \int \psi_\lambda(\underline{x}')\, \hat{\psi}^+(\underline{x}')\, d\tau'$$

$$= \iint d\tau\, d\tau' \sum_{\lambda=1}^{\infty} \psi_\lambda(\underline{x}')\, \psi_\lambda^*$$

$$\hat{N} = \sum_{\lambda=1}^{\infty} a_\lambda^+ a_\lambda = \sum_{\lambda=1}^{\infty} \int \psi_\lambda(\underline{x})\, \hat{\psi}^+(\underline{x})\, d\tau \int \psi_\lambda^*(\underline{x}')\, \hat{\psi}(\underline{x}')\, d\tau'$$

$$= \int d\tau \int d\tau' \underbrace{\sum_{\lambda=1}^{\infty} \psi_\lambda(\underline{x})\, \psi_\lambda^*(\underline{x}')}_{\delta(\underline{x}-\underline{x}')} \hat{\psi}^+(\underline{x})\, \hat{\psi}(\underline{x}') = \int d\tau\, d\tau'\, \delta(\underline{x}-\underline{x}')\, \hat{\psi}^+(\underline{x})\, \hat{\psi}(\underline{x}')$$

$$= \int \hat{\psi}^+(\underline{x})\, \hat{\psi}(\underline{x})\, d\tau \Rightarrow \int \hat{n}(\underline{x})\, d\tau\,, \quad \hat{n}(\underline{x}) = \hat{\psi}^+(\underline{x})\, \hat{\psi}(\underline{x})$$

$$\hat{H} = \sum_{\lambda,\mu}^{1\ldots\infty} a_\lambda^+ A_{\lambda\mu} a_\mu \,, \qquad A_{\lambda\mu} = \langle \psi_\lambda | A | \psi_\mu \rangle = \int \psi_\lambda^*(\underline{x})\, A(\underline{x})\, \psi_\lambda(\underline{x})\, d\tau$$

$$= \sum_{\lambda,\mu}^{1\ldots\infty} a_\lambda^+ \int \psi_\lambda^*(\underline{x})\, A(x)\, \psi_\mu(\underline{x})\, d\tau\; a_\mu$$

$$= \int \underbrace{\sum_{\lambda=1}^{\infty} \left(\psi_\lambda^*(x)\, a_\lambda^+ \right)}_{\hat{\psi}^+(\underline{x})} A(\underline{x}) \underbrace{\sum_{\mu=1}^{\infty} \psi_\mu(\underline{x})\, a_\mu}_{\hat{\psi}(x)} = \int \hat{\psi}^+(x)\, A(x)\, \hat{\psi}(x)\, d\tau$$

z. B. Kinetische Energie ohne Spin $A(\underline{x}) = -\frac{\hbar^2}{2m}\Delta$

$$\Rightarrow \hat{H} = \int \hat{\psi}^+(\vec{r}) \left(-\frac{\hbar^2}{2m}\Delta \right) \hat{\psi}(\vec{r})\, d^3r \,, \qquad \hat{H}\, |\vec{n}_1 \vec{n}_1 \ldots\rangle = \ldots$$

$$\left[\hat{\psi}(\underline{x}), \hat{A} \right] = \left[\hat{\psi}(\underline{x}) \int \hat{\psi}^+(\underline{x}')\, A(\underline{x}',t)\, \hat{\psi}(\underline{x}')\, d\tau' \right]$$

$$= \int \hat{\psi}(\underline{x})\, \hat{\psi}^+(\underline{x}')\, A(\underline{x}',t)\, \hat{\psi}(\underline{x}')\, d\tau' - \int \hat{\psi}^+(\underline{x}')\, A(\underline{x}',t)\, \hat{\psi}(\underline{x}')\, \hat{\psi}(\underline{x})$$

$$= \int \hat{\psi}^+(\underline{x}')\, \hat{\psi}(\underline{x})\, A(\underline{x}',t)\, \hat{\psi}(\underline{x}')\, d\tau' + \int \delta(\underline{x}-\underline{x}')\, A(\underline{x}',t)\, \hat{\psi}(\underline{x}')\, d\tau' -$$

$$- \int \hat{\psi}^+(\underline{x}')\, A(\underline{x}',t)\, \hat{\psi}(\underline{x}')\, \hat{\psi}(\underline{x})\, d\tau'$$

$$= \int \hat{\psi}^+(\underline{x}')\, \cancel{A(\underline{x}',t)\, \hat{\psi}(\underline{x}')}\, \hat{\psi}(\underline{x})\, d\tau' + A(\underline{x},t)\, \hat{\psi}(x) - \int \hat{\psi}^+(\underline{x}')\, \cancel{A(\underline{x}',t)\, \hat{\psi}(\underline{x}')}\, d\tau'\, \hat{\psi}(x)$$

$$= A(x,t)\, \hat{\psi}(\underline{x},t)$$

$$\boxed{\varrho = U \varrho_0 U^+}$$

$$\dot{\varrho} = \dot{U} \varrho_0 U^+ + U \varrho_0 \dot{U}^+$$

$$= -\frac{i}{\hbar} H U \varrho_0 U^+ + U \varrho_0 \frac{i}{\hbar} U^+ H$$

$$= -\frac{i}{\hbar} H \varrho + \frac{i}{\hbar} \varrho H = \frac{i}{\hbar} [\varrho, H]$$

$$U^+ = \exp\left\{ \frac{i}{\hbar} H (t-t_0) \right\}$$

$$\dot{U} = -\frac{i}{\hbar} H U$$

$$\dot{U}^+ = \frac{i}{\hbar} H U^+ = \frac{i}{\hbar} U^+ H$$