5.2 Feldoperatoren

Bosonen

$$[\hat\psi(\underline{x}), \hat\psi^+(\underline{x}')] =$$

$$= \left[\sum_{\nu=1}^{\infty} \psi_\nu(\underline{x})\, a_\nu, \sum_{\mu=1}^{\infty} \psi^*_\mu(\underline{x}')\, a^+_\mu\right] = \sum_{\nu,\mu}^{1\ldots\infty} \psi_\nu(\underline{x})\, \psi^*_\mu(\underline{x}')\, [a_\nu, a^+_\mu]$$

$$= \sum_{\nu,\mu}^{1\ldots\infty} \psi_\nu(\underline{x})\, \psi^*_\mu(\underline{x}')\, \delta_{\nu\mu}\, 1 = \sum_{\nu=1}^{\infty} \psi_\nu(\underline{x})\, \psi^*_\nu(\underline{x}')\, \underline{1} = \delta(\underline{x} - \underline{x}')\, \underline{1}$$

$$\hat N = \sum_{\lambda=1}^{\infty} a^+_\lambda a_\lambda = \sum_{\lambda=1}^{\infty} \int \psi^*_\lambda(\underline{x})\, \hat\psi(\underline{x})\, d\tau \int \psi_\lambda(\underline{x}')\, \hat\psi^+(\underline{x}')\, d\tau'$$

$$= \iint d\tau\, d\tau' \sum_{\lambda=1}^{\infty} \psi_\lambda(\underline{x}')\, \psi^*_\lambda$$

$$\hat N = \sum_{\lambda=1}^{\infty} a^+_\lambda a_\lambda = \sum_{\lambda=1}^{\infty} \int \psi_\lambda(\underline{x})\, \hat\psi^+(\underline{x})\, d\tau \int \psi^*_\lambda(\underline{x}')\, \hat\psi(\underline{x}')\, d\tau'$$

$$= \int d\tau \int d\tau' \underbrace{\sum_{\lambda=1}^{\infty} \psi_\lambda(\underline{x})\, \psi^*_\lambda(\underline{x}')}_{\delta(\underline{x}-\underline{x}')} \hat\psi^+(\underline{x})\, \hat\psi(\underline{x}') = \int d\tau\, d\tau'\, \delta(\underline{x}-\underline{x}')\, \hat\psi^+(\underline{x})\, \hat\psi(\underline{x}')$$

$$= \int \hat\psi^+(\underline{x})\, \hat\psi(\underline{x})\, d\tau \Rightarrow \int \hat n(\underline{x})\, d\tau\,, \quad \hat n(\underline{x}) = \hat\psi^+(\underline{x})\, \hat\psi(\underline{x})$$

$$\hat H = \sum_{\lambda,\mu}^{1\ldots\infty} a^+_\lambda A_{\lambda\mu} a_\mu\,, \qquad A_{\lambda\mu} = \langle \psi_\lambda | A | \psi_\mu \rangle = \int \psi^*_\lambda(\underline{x})\, A(\underline{x})\, \psi_\mu(\underline{x})\, d\tau$$

$$= \sum_{\lambda,\mu}^{1\ldots\infty} a^+_\lambda \int \psi^*_\lambda(\underline{x})\, A(x)\, \psi_\mu(\underline{x})\, d\tau\, a_\mu$$

$$= \int \underbrace{\sum_{\lambda=1}^{\infty} \left(\psi_\lambda^*(x)\, a_\lambda^+ \right)}_{\hat\psi^+(\underline{x})} A(\underline{x}) \underbrace{\sum_{\mu=1}^{\infty} \psi_\mu(\underline{x})\, a_\mu}_{\hat\psi(\underline{x})} = \int \hat\psi^+(x)\, A(x)\, \hat\psi(x)\, d\tau$$

z.B. Kinetische Energie ohne Spin $A(\underline{x}) = -\frac{\hbar^2}{2m}\Delta$

$$\Rightarrow \hat H = \int \hat\psi^+(\vec r)\left(-\frac{\hbar^2}{2m}\Delta\right)\hat\psi(\vec r)\, d^3r \quad , \quad \hat H\, |\vec r_1\, \vec r_2, \ldots\rangle = \ldots\,.$$

$$\left[\hat\psi(\underline{x}), \hat A\right] = \left[\hat\psi(\underline{x}) \int \hat\psi^+(\underline{x}')\, A(\underline{x}',t)\, \hat\psi(\underline{x}')\, d\tau'\right]$$

$$= \int \hat\psi(\underline{x})\, \hat\psi^+(\underline{x}')\, A(\underline{x}',t)\, \hat\psi(\underline{x}')\, d\tau' - \int \hat\psi^+(\underline{x}')\, A(\underline{x}',t)\, \hat\psi(\underline{x}')\, \hat\psi(\underline{x})$$

$$= \int \hat\psi^+(\underline{x}')\, \hat\psi(\underline{x})\, A(\underline{x}',t)\, \hat\psi(\underline{x}')\, d\tau' + \int \delta(\underline{x}-\underline{x}')\, A(\underline{x}',t)\, \hat\psi(\underline{x}')\, d\tau' -$$

$$- \int \hat\psi^+(\underline{x}')\, A(\underline{x}',t)\, \hat\psi(\underline{x}')\, \hat\psi(\underline{x})\, d\tau'$$

$$= \int \hat\psi^+(\underline{x}')\, \cancel{A(\underline{x}',t)\, \hat\psi(\underline{x}')}\, \hat\psi(\underline{x})\, d\tau' + A(\underline{x},t)\, \hat\psi(x) - \int \hat\psi^+(\underline{x}')\, \cancel{A(\underline{x}',t)\, \hat\psi(\underline{x}')}\, d\tau'\, \hat\psi(x)$$

$$= A(x,t)\, \hat\psi(\underline{x},t)$$

$$\boxed{\varrho = U \varrho_0 U^+}$$

$$\dot\varrho = \dot U \varrho_0 U^+ + U \varrho_0 \dot U^+$$

$$U^+ = \exp\left\{\frac{i}{\hbar} H (t-t_0)\right\}$$

$$\dot U = -\frac{i}{\hbar} H U$$

$$\dot U^+ = \frac{i}{\hbar} H U^+ = \frac{i}{\hbar} U^+ H$$

$$= -\frac{i}{\hbar} H U \varrho_0 U^+ + U \varrho_0 \frac{i}{\hbar} U^+ H$$

$$= -\frac{i}{\hbar} H \varrho + \frac{i}{\hbar} \varrho H = \frac{i}{\hbar}[\varrho, H]$$