6.4 Kohärente Zustände

$$\hat{\vec{E}} = \frac{i}{\sqrt{2}}\sqrt{\frac{\hbar\omega}{\varepsilon V}}\,\vec{u}\left[\exp\{i(\vec{q}\cdot\vec{r}-\omega t)\}c - \exp\{-i(\vec{q}\cdot\vec{r}-\omega t)\}c^+\right]$$

$$c|\alpha\rangle = \alpha|\alpha\rangle, \quad \alpha\in\mathbb{C}, \quad \alpha = |\alpha|\exp\{i\varphi\}, \quad \langle\alpha|\alpha\rangle = 1$$

$$\Rightarrow \alpha = \langle\alpha|c|\alpha\rangle = \langle c^+\alpha|\alpha\rangle = \langle\alpha|c^+|\alpha\rangle^* \Rightarrow \langle\alpha|c^+|\alpha\rangle = \alpha^*$$

$$c^+|\alpha\rangle = \alpha^*|\alpha\rangle$$

$$\Rightarrow \langle\alpha|\hat{\vec{E}}|\alpha\rangle = \frac{i}{\sqrt{2}}\sqrt{\frac{\hbar\omega}{\varepsilon V}}\,\vec{u}\,|\alpha|\left[\exp\{i(\vec{q}\cdot\vec{r}-\omega t+\varphi\}-\exp\{-i(\vec{q}\cdot\vec{r}-\omega t+\varphi)\}\right]$$

$$= -\sqrt{2}\sqrt{\frac{\hbar\omega}{\varepsilon V}}\,\vec{u}\,|\alpha|\sin(\vec{q}\cdot\vec{r}-\omega t+\varphi)$$

$$\exp\{a\}\exp\{b\} = \exp\{a+b\} = \exp\{b\}\exp\{a\} \quad \text{nur bei } [a,b]=0$$

Wenn $[a,b] = x$ <u>und</u> $[a,x] = 0 = [b,x]$

$$[a,b^n] = n\,b^{n-1}x$$

vollständige Induktion

$$[a,b^{n+1}] = ab^nb - b^nba = b^nab + nb^{n-1}xb - b^nba$$

$$= \cancel{b^nba} + b^nx + nb^nx - \cancel{b^nba}$$

$$= (n+1)b^nx$$

$$[a,\exp\{b\}] = x\exp\{b\} \Rightarrow \exp\{b\}a = (a-x)\exp\{b\}$$

$$\Rightarrow \exp\{b\}a^n = (a-x)^n\exp\{b\} \quad \text{v.I.}$$

$$\exp\{b\}a^{n+1} = (a-x)\exp\{b\}a^n = (a-x)(a-x)^n\exp\{b\} = (a-x)^{n+1}\exp\{b\}$$