Reaction-Diffusion systems in Biology

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1. Introduction

Synergetics, Self-organization, Complexity.

2. Linear systems

First order equations
- *Simple example, logistic population model, constant haversting*

Planar linear systems
- *Second orden differential equations, planar linear systems, eigenvalues and eigenvectors, solving linear systems*

Phase portrait
- *Real distinct eigenvalues, complex eigenvalues, repeated eigenvalues, change of coordinates*

Clasification of planar systems

3. Biochemical Reactions

Law of mass action
- *Simple reaction, Bimolecular reaction, dimerization, Trimolecular reaction*

Enzyme kinetics
- *Michaelis-Menten dynamics, cooperativity, Hill function*

Goldbeter-Koshland mechanism
- *Phosphorylation, dephosphorylation, switches*

4. **Diffusion**

Microscopic theory
- *Brownian motion, random walk, diffusion coefficient*

Macroscopic theory
- *First Fick's law, Second Fick's law, Example in 1D*

Typical values of Diffusion coefficients
- *Einstein's relation, characteristic diffusion values, diffusion coefficients inside cells*

5. Reaction-diffusion

Reaction-Diffusion equations
- *Conservation law, travelling wave solution*

Fischer-Kolmogorov equation
- *Propagation into a unstable state, continuous spectra of velocities*

Schlögl equation
- *Propagation into a metastable state, single velocity*

6. Mathematical aspects

Brusselator model
- *History, mass action law*

Temporal dynamics
- *Stability analysis, Hopf bifurcation*

Spatio temporal dynamics
- *Stability analysis, Turing bifurcation, Co-dimension two bifurcation*

7. Chemical waves

CO oxidation in catalytic surfaces
- *Mechanism, KEE model*

BZ reaction
- *History, oscillations, FKN model, Oregonator model, exicitability, photosensitive version, waves*

8. Integration methods of PDE

Temporal integration of ODE
- *Taylor series, Eiler method, Runge-Kutta method*

Spatial finite differences
- *Forward, backward, centered formula*

Spatio-temporal methods
- *Euler method, Crank-Nicholson method*

Boundary and initial conditions

9. Membrane potential propagation

Nerst potential
- *Membrane potential, Nerst-Planck equation*

Ion channels
- *Electric circuit, channel gating*

Hodgkin-Huxley model
- *Sodium current, potassium current, oscillations*

Cable equation

10. Excitability

Membrane potential
- *Hodgkin-Huxley model, simpler version*

FitzHugh-Nagumo model
- *Nullclines, excitability, bistability, oscillatory*

Waves in extended systems
- *Traveling wave solution, curvature effects, spiral waves, scroll waves*

11. Glycolysis

Glycolosys
- *Metabolism, Glucose oxidation, reactions*

Sel´kov model
- *Model derivation, quasi-steady approximation, oscillations*

Extended Sel´kov model
- *Model derivation, spatial waves*

12. Calcium waves

Calcium dynamics
- *Oscillations, Waves*

Calcium cell models
- *Influx, Mitocondria, Calcium Buffers, Calcium pumps, IP3 dynamics*

Calcium Waves
- *Spiral waves, traveling waves*

13. Cell polarization

Cell division
- *E. Coli, C. Elegans*

Cell motion
- *Chemotaxis, Dictyostelium discoideum, Neutrophils, LEGI, bistability*

Generic mass-conserved models

14. Cardiac tissue

Heart
- *Wave propagation, Contration, Myocytes, Tissue, ECG, Fibrillation*

Myocytes
- *Membrane action potential, ion chanels*

Tissue
- *1D fibers, Bidomain model, monodomain*

Bibliography

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H. C. Berg, Random walks in Biology (Princeton).