**Stochastic Differential Equations in Applications**

## TUTORIAL 1: Probability, random walk and mean exit time

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- 1. Using two independent uniformly distributed random numbers $(u_1, u_2) \in ([0,1],[0,1])$ generate a pair $(p_x, p_y)$ with the relativistic Jüttner distribution density $\rho(p_x, p_y) \sim \exp\left(-\frac{\sqrt{p_x^2+p_y^2}}{kT}\right)$.
- 2. Determine the distribution of the absolute relative velocity $u = \mid \vec{u}_1 - \vec{u}_2 \mid$ in a gas of active particles.
- 3. Determine the pressure in a gas of active particles with the density $\rho_0$ and the absolute velocity $v_0$.
- 4. Determine the conditional distribution density $p(i,t|0,0)$ for a symmetric walker on a circle with $N$ sites.
- 5. Random walkers are released at $x = a > 0$ at the rate $R$ walkers per unit of time. The walkers are symmetric and move in a one-dimensional domain, bounded by a wall at $x = b > a$ and by a sink at $x = 0$. Consider the problem in the continuous limit. Assuming that the diffusion coefficient $D$ is known, determine the average number of walkers in the domain in the stationary state.
- 6. Determine the mean exit time for an asymmetric walker that moves in the domain bounded by a sink at $x = 0$ and by a reflecting wall at $x = b > 0$. The walker starts the motion in $x_0$, $(0 < x_0 < b)$.