5.3 Mobility and spread of infectious diseases

- How can mobility be measured/quantified? \( \text{meas} \) of transportation activity...
- What kind of dynamics/behavior can be expected? \( \text{diffusion, random walk} \)

**Idea:** use proxy data \( \text{account of bills (online platform)} \)
- probability for distance \( r: p(r) \sim \frac{1}{r^\mu} \)
  \( \mu = 2 \)
- typical distance \( 1x(t) \sim r^2 \) with \( \mu < 2 \) (faster than random walk)
- description via Lévy flights (jump distance according to \( p(r) \))
- combination with susceptible-infected-susceptible model

\[
\begin{align*}
S_1 & \rightarrow T_1, \quad I_1 \rightarrow S_1
\end{align*}
\]

- average waiting time: \( <\tau> = \frac{1}{2 \mu \lambda_m} \)
- conserved population size: detailed balance:
  \( \omega_{m,n} \left( \frac{S_m I_n}{S_m + I_n} \right) = \omega_{m,n} \left( \frac{S_{m+1} I_n}{S_{m+1} + I_n} \right) \)

- long-distance jumps trigger reduction of distant infection points
  \( \Rightarrow \text{diffusion from diffusion-like spreading} \)
  \( \Rightarrow \text{comparision to empirical data (SARS)} \)

5.4 Modellheit in (sozialen) Netzwerken

**Aufgabe:** Teile Netzwerk in Gruppen (Communities), die sich stark als zwischen einzelnen verknüpft sehen.

Idee: Vergleiche mit zu erwartenden Links zwischen innerhalb von Gruppen in einem vollständigen Netzwerk:

\[
\begin{align*}
\frac{k_i k_j}{2L} \quad \text{Wahrscheinlichkeit, dass bei Linken Kunden in} \\
\text{gleiche Gruppe} & \quad \text{mit Grad } k_i \text{ und Kunden mit Grad } k_j \\
\text{#Kunden} & \quad \text{verknüpft sind}.
\end{align*}
\]
Modularity and community structure in networks

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Many networks of interest in the sciences, including social networks, computer networks, and metabolic and regulatory networks, are found to divide naturally into communities or modules. The problem of detecting and characterizing this community structure is one of the outstanding issues in the study of networked systems. One highly effective approach is the optimization of the quality function known as “modularity” over the possible divisions of a network. Here I show that the modularity can be expressed in terms of the eigenvectors of a characteristic matrix for the network, which I call the modularity matrix, and that this expression leads to a spectral algorithm for community detection that returns results of demonstrably higher quality than competing methods in shorter running times. I illustrate the method with applications to several published network data sets.

clustering | partitioning | modules | metabolic network | social network

Many systems of scientific interest can be represented as networks, sets of nodes or vertices joined in pairs by lines...
Suppose our network contains \( n \) vertices. For a particular division of the network into two groups let \( s_i = 1 \) if vertex \( i \) belongs to group 1 and \( s_i = -1 \) if it belongs to group 2. And let the number of edges between vertices \( i \) and \( j \) be \( A_{ij} \), which will normally be 0 or 1, although larger values are possible in networks where multiple edges are allowed. (The quantities \( A_{ij} \) are the elements of the so-called adjacency matrix.) At the same time, the expected number of edges between vertices \( i \) and \( j \) if edges are placed at random is \( k_i k_j / 2m \), where \( k_i \) and \( k_j \) are the degrees of the vertices and \( m = \frac{1}{2} \Sigma_i k_i \) is the total number of edges in the network. Thus the modularity \( Q \) is given by the sum of 
\[
A_{ij} - k_i k_j / 2m 
\]
over all pairs of vertices \( i,j \) that fall in the same group.

Observing that the quantity \( \frac{1}{2} (s_i s_j + 1) \) is 1 if \( i \) and \( j \) are in the same group and 0 otherwise, we can then express the modularity as

\[
Q = \frac{1}{4m} \sum_{ij} \left( A_{ij} - \frac{k_i k_j}{2m} \right) (s_i s_j + 1) = \frac{1}{4m} \sum_{ij} \left( A_{ij} - \frac{k_i k_j}{2m} \right) s_i s_j, \tag{1}
\]

where the second equality follows from the observation that 
\[
2m = \Sigma_i k_i = \Sigma_{ij} A_{ij}. \quad \text{The leading factor of } 1/4m \text{ is merely}
\]

Eq. 1 can conveniently be written in matrix form as

\[
Q = \frac{1}{4m} s^T B s, \tag{2}
\]

where \( s \) is the column vector whose elements are the \( s_i \) and we have defined a real symmetric matrix \( B \) with elements

\[
B_{ij} = A_{ij} - \frac{k_i k_j}{2m}, \tag{3}
\]
Fig. 2. Application of the eigenvector-based method to the karate club network of ref. 23. Shapes of vertices indicate the membership of the corresponding individuals in the two known factions of the network, and the dotted line indicates the split found by the algorithm, which matches the factions exactly. The shades of the vertices indicate the strength of their membership, as measured by the value of the corresponding elements of the eigenvector.

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Vincent Blondel, Gautier Krings, Isabelle Thomas

Regions and borders of mobile telephony in Belgium and in the Brussels metropolitan zone
Anrede ausgehend von/eingebettet nach Arlon (Wallonie)

Ostende (Flamand)

Community detection bei 4 Gruppen (ohne Kontrolle des Ortes)

Die Landkarte projiziert folgt der Trennung der Sprachgruppen

200 Millionen Kontakte, Nord → Süd 1.05% 98% der Kommunikation
Süd → Nord 1.04% innerhalb der Gruppe

2) Tierhandel

Daraus aus EU-Richtlinie 1) Verpflichtung der Mitgliedstaaten, Daraus zum Tierhandel
zu achten (Schweine, Rinder, Schafe, Pferde...)

Deutschland: HI-Tier (Herkunftssicherungs- und Informationsystem Tier)
Ziel: Nachvollziehbarkeit von Zugängen/Abgängen + Gesundheitszustand

→ Bei fachlicher Einleitung von Gegenmaßnahmen
Pflege/Eintragung in Datenbank durch landwirtschaftliche Betriebe in Anzahl von 7 Tagen

→ Kontaktnetzwerk: Kuhbetrieb → Betrieben
Kuhbetrieb → Handelskontakts

Beisp. Schwerinland: ca. 100.000 Kuhbetriebe, 200.000 Kuhbetriebe pro Jahr

→ strukturierte Produktionskette, Lebenslauf 180 Tagen

Frage: Struktur des Hauses??
Number of premises and number of trade connections within the trade communities. The community IDs are arbitrarily chosen.

<table>
<thead>
<tr>
<th>Community ID</th>
<th>Number of premises</th>
<th>Number of trade edges</th>
</tr>
</thead>
<tbody>
<tr>
<td>51</td>
<td>26,138</td>
<td>81,768</td>
</tr>
<tr>
<td>24</td>
<td>25,965</td>
<td>81,756</td>
</tr>
<tr>
<td>9</td>
<td>19,923</td>
<td>46,546</td>
</tr>
<tr>
<td>43</td>
<td>14,495</td>
<td>30,847</td>
</tr>
<tr>
<td>61</td>
<td>11,420</td>
<td>20,455</td>
</tr>
<tr>
<td>83</td>
<td>10,496</td>
<td>20,441</td>
</tr>
<tr>
<td>17</td>
<td>4772</td>
<td>8836</td>
</tr>
<tr>
<td>101</td>
<td>1222</td>
<td>1606</td>
</tr>
<tr>
<td>132</td>
<td>660</td>
<td>783</td>
</tr>
<tr>
<td>Others</td>
<td>4767</td>
<td>5231</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>119,858</strong></td>
<td><strong>298,269</strong></td>
</tr>
</tbody>
</table>

![Spatial distribution and overlap of the trade communities. Left: dots represent premises at their approximate locations. The coloring indicates their community membership. Right: regions of spatial overlap. The areas shown contain 95% of the premises of each cluster. The cluster IDs are: grey: 24, light blue: 17, red: 9, dark blue: 132, orange: 83, purple: 51, green: 101, pink: 61 and yellow: 43. The cluster IDs are arbitrarily chosen.](image-url)
Fig. 2. Network representation of the nine largest trade communities of the German pig production network. The size of the nodes corresponds to the number of intra-community trade connections and the numbers are community IDs (see Table 1). The widths of the edges correspond to the number of trade contacts between communities, e.g., the maximum value is 7231 trade connections between communities 24 and 9. Dashed lines represent edges with less than 100 trade contacts. The position of the nodes approximately reflects the geographical locations of the trade clusters in Fig. 1.