English summary

3. Physiological models

3.1 Hodgkin–Huxley model

first principles: physical, electrical, and chemical fundamentals such as Ohm's law, Kirchhoff's junction rule, Nernst potential

$$c_m \dot{V} = -\bar{g}_K n^4 (V - E_K) - \bar{g}_{Na} m^3 h (V - E_{Na}) - \bar{g}_L (V - E_L)$$

E_K, E_{Na}, E_L: reversal potentials of K, Na, L

membrane potential

$\bar{g}_K, \bar{g}_{Na}, \bar{g}_L$: (maximum) conductances of K, Na, leak channels

$n(V)$: gating variable (K activation) $\quad \dot{n} = \alpha_n(V)(1-n) - \beta_n(V)\, n$

$m(V)$: " (Na activation) $\quad \dot{m} = \alpha_m(V)(1-m) - \beta_m(V)\, m$

$h(V)$: " (Na inactivation) $\quad \dot{h} = \alpha_h(V)(1-h) - \beta_h(V)\, h$

or equivalently:

$$\tau_n(V)\,\dot{n} = -n + n_\infty(V)$$

$$\tau_m(V)\,\dot{m} = -m + m_\infty(V)$$

$$\tau_h(V)\,\dot{h} = -h + h_\infty(V)$$

$$\alpha_n(V) = 0.01\frac{10 - V}{e^{(10-V)/10} - 1}$$

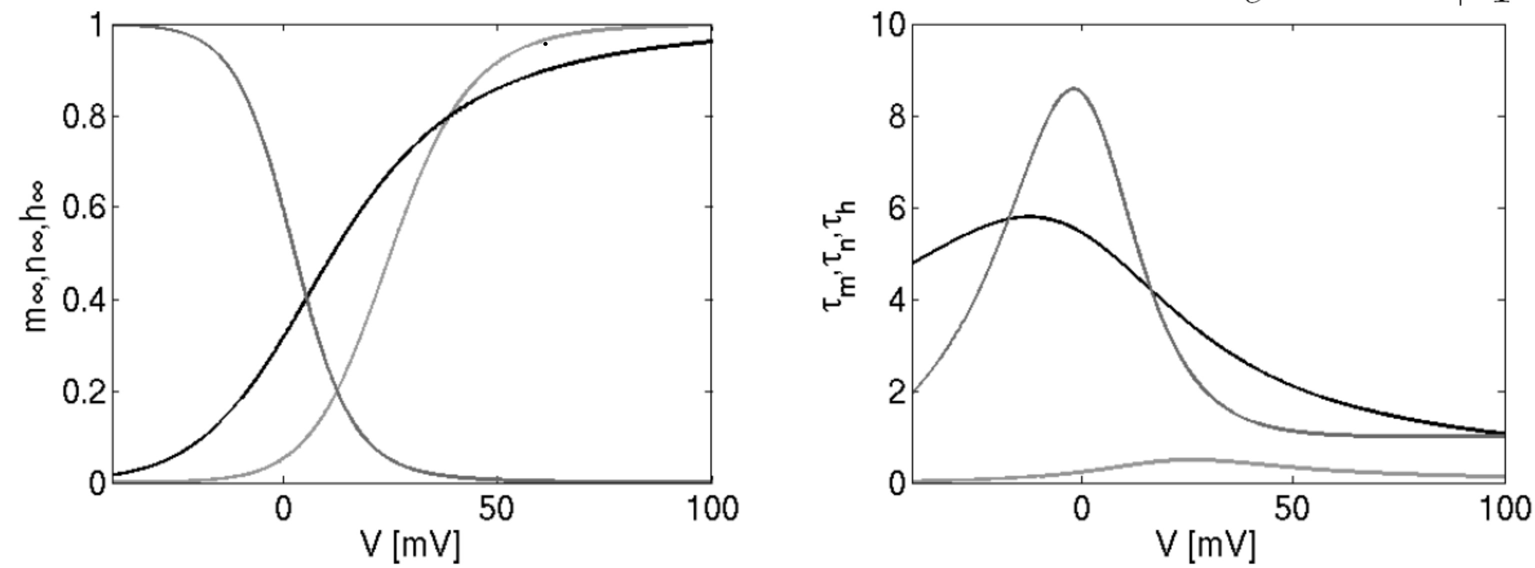
$$\beta_n(V) = 0.125 e^{-V/80}.$$

$$\alpha_m(V) = 0.1\frac{25 - V}{e^{(25-V)/10} - 1}$$

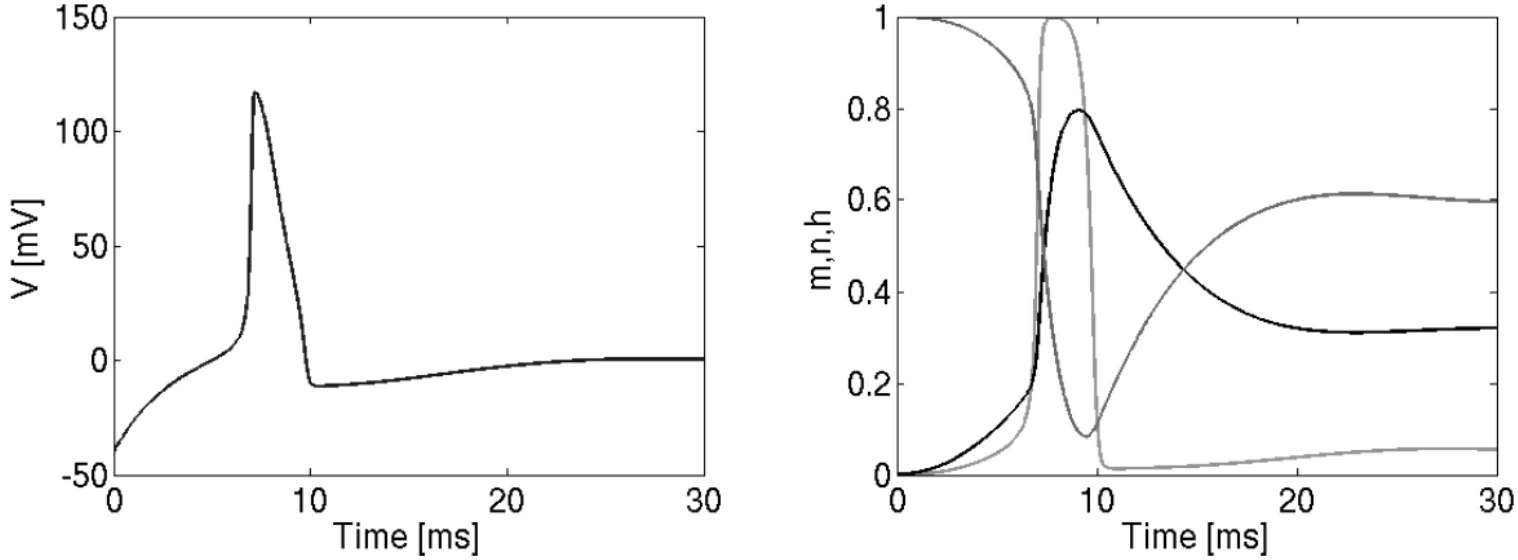
$$\beta_m(V) = 4e^{-V/18}$$

$$\alpha_h(V) = 0.07e^{-V/20}$$

$$\beta_h(V) = \frac{1}{e^{(30-V)/10} + 1}.$$



Left panel:Steady state activation and inactivaion functions. Right panel: Voltage dependent time constants, for sodium activation m (green) and inactivation h (pink) and potassium activation n (black).



Left panel: Membrane voltage. Right panel: Activation potential, for sodium activation m (green) and inactivation h (pink) and potassium activation n (black).

3.2 Morris-Lecar model

BIOPHYS. J. © Biophysical Society · 0006–3495/81/07/193/21 $1.00 193
Volume 35 July 1981 193–213

VOLTAGE OSCILLATIONS IN THE BARNACLE GIANT MUSCLE FIBER

CATHERINE MORRIS AND HAROLD LECAR, *Laboratory of Biophysics, National Institute of Neurological and Communicative Disorders and Stroke, National Institutes of Health, Bethesda, Maryland 20205*

2-Variablenmodell:

$$C_m \dot{V} = I - g_L (V - E_L) - \underbrace{g_{Ca}\, m_\infty(V)(V - E_{Ca})} - g_K\, w(V)(V - E_K)$$

Bifurkationsparameter
zusätzlicher Strom

Calcium immer im Gleichgewicht
(Fixpunkt) aufgrund schneller Dynamik

$\tau_m \dot{m} = -m + m_\infty(V)$

$\dot{m} \overset{!}{=} 0 \Rightarrow m = m_\infty(V)$

$$\tau_w(V)\, \dot{w} = -w + w_\infty(V)$$

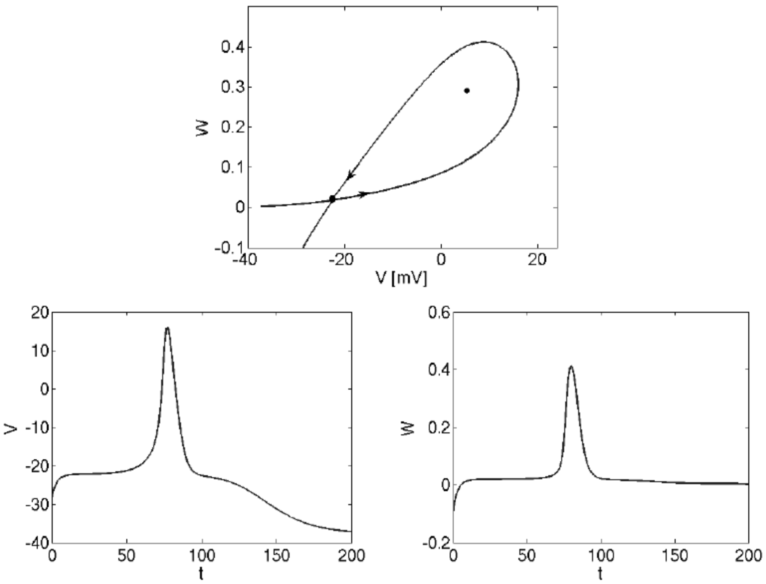
Parameter (Potenzial abhängig):

$$m_\infty(V) = \frac{1}{2}\left[1+\tanh\frac{V-V_1}{2}\right] = \left[1+\exp\left(-2\,\frac{V-V_1}{V_2}\right)\right]^{-1}$$

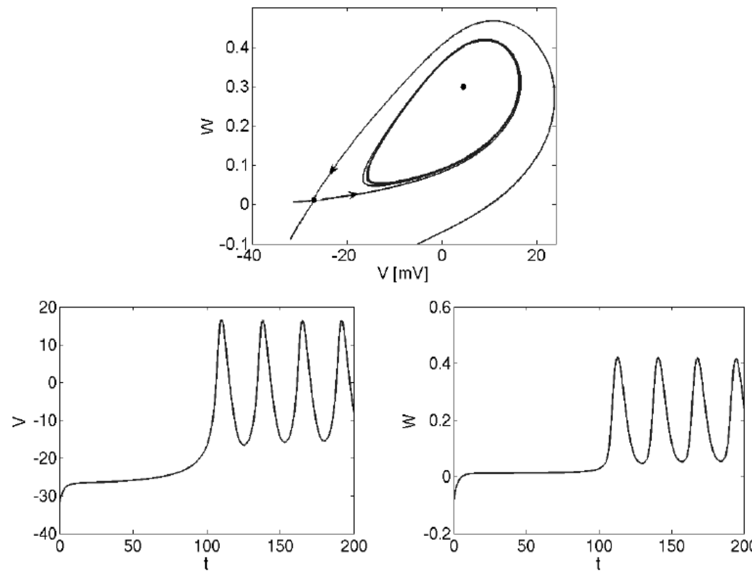
$$w_\infty(V) = \frac{1}{2}\left[1+\tanh\frac{V-V_3}{V_4}\right] = \left[1+\exp\left(-2\,\frac{V-V_3}{V_4}\right)\right]^{-1}$$

$$\tau_w(V) = \frac{1}{\cosh\frac{V-V_3}{2V_4}}$$

V_1, V_2, V_3, V_4: Parameter zum Einstellen des Fixpunkts

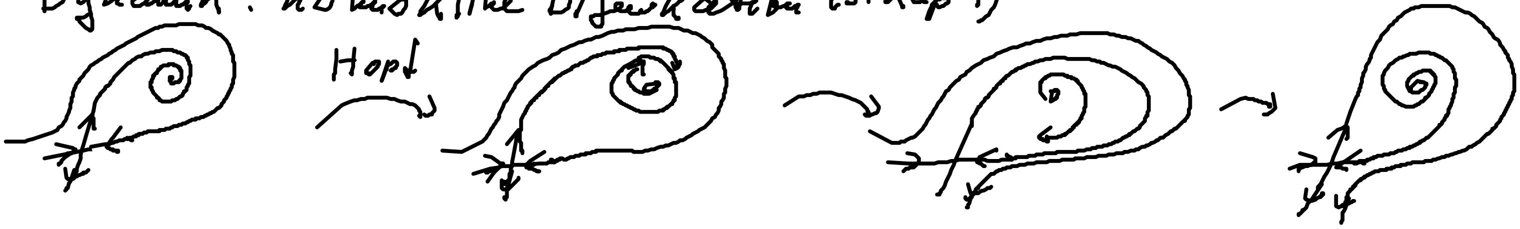


The phase portrait of the homoclinic bifurcation present in the Morris-Lecar model at $I_0 = 35$ and corresponding time series for v and ω.



The phase portrait of Morris-Lecar at $I_0 = 39.5$ (oscillatory regime) and corresponding time series for v and ω.

Dynamik: homokline Bifurkation (s. Kap 1)



4. Wechselspiel von Rauschen und Zeitverzögerung

bisher: deterministische dynamische Systeme, jetzt: stochastische dynamische Systeme

4.1 Kohärenzresonanz

4.2 zeitverzögerte Rückkopplung

Zeitverzögerungen in gekoppelten Systemen in Kapiteln 5 und 6

4.1 Kohärenzresonanz

- Konstruktiver Einfluss von Rauschen
  ⇒ Regularität rauschinduzierter Oszillationen (optimale Rauschstärke)

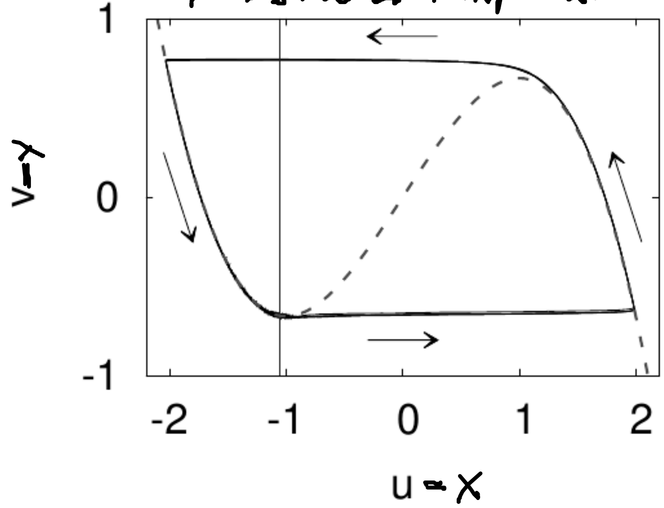
Bsp.: FitzHugh-Nagumo-System mit Rauschen

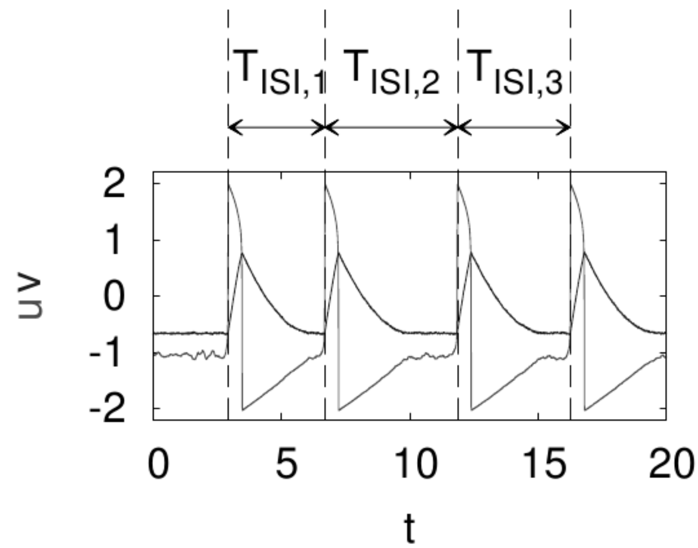
$$\varepsilon \dot{x} = x - \frac{x^3}{3} - y$$

$$\dot{y} = x + a \underbrace{+ D\xi(t)}_{\text{additives Rauschen}}$$

rauschinduzierte Oszillationen
(Schwankungen in der Periode T_{ISI})

$a>1 \Rightarrow$ stabiles Fixpunkt

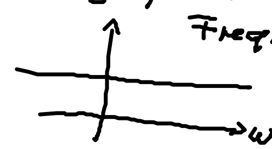




$\xi(t)$: Gauß'sches weiße Rauschen

spektrale Leistungsdichte: $S(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \langle \xi(t)\,\xi(t+s)\rangle\, e^{i\omega s}\, ds = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(s)\, e^{i\omega s}\, ds = \frac{1}{2\pi}$

$\langle \xi(t)\,\xi(t')\rangle = \delta(t-t')$ unkorreliert



keine bevorzugte Frequenz $\Rightarrow$ weiß

VOLUME 78, NUMBER 5 PHYSICAL REVIEW LETTERS 3 FEBRUARY 1997

Coherence Resonance in a Noise-Driven Excitable System

Arkady S. Pikovsky* and Jürgen Kurths*

Max–Planck–Arbeitsgruppe "Nichtlineare Dynamik" an der Universität Potsdam Am Neuen Palais 19, PF 601553, D-14415, Potsdam, Germany

(Received 9 August 1996)

We study the dynamics of the excitable Fitz Hugh–Nagumo system under external noisy driving. Noise activates the system producing a sequence of pulses. The coherence of these noise-induced oscillations is shown to be maximal for a certain noise amplitude. This new effect of coherence resonance is explained by different noise dependencies of the activation and the excursion times. A simple one-dimensional model based on the Langevin dynamics is proposed for the quantitative description of this phenomenon. [S0031-9007(97)02349-1]

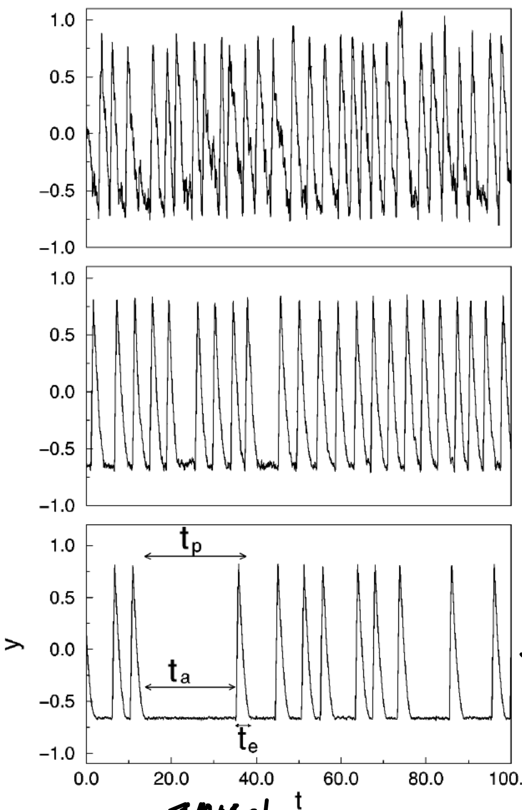
Maße zum Quantifizieren der Regularität

- Korrelationszeit: $t_{cor} = \frac{1}{\psi(0)} \int_0^\infty |\psi(s)|\, ds$, $\psi(0) = \langle [x(t)-\langle x\rangle][x(t)-\langle x\rangle]\rangle = \langle [x(t)-\langle x\rangle]^2\rangle$ Varianz

  Autokorrelationsfunktion: $\psi(s) = \langle [x(t)-\langle x\rangle][x(t-s)-\langle x\rangle]\rangle$

  $\langle x\rangle = 0$: $= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} dt\; x(t)\, x(t+s)$

- normierte Fluktuationen der Interspikeintervalle: $R = \frac{\sqrt{\langle T_{ISI}^2\rangle - \langle T_{ISI}\rangle^2}}{\langle T_{ISI}\rangle}$



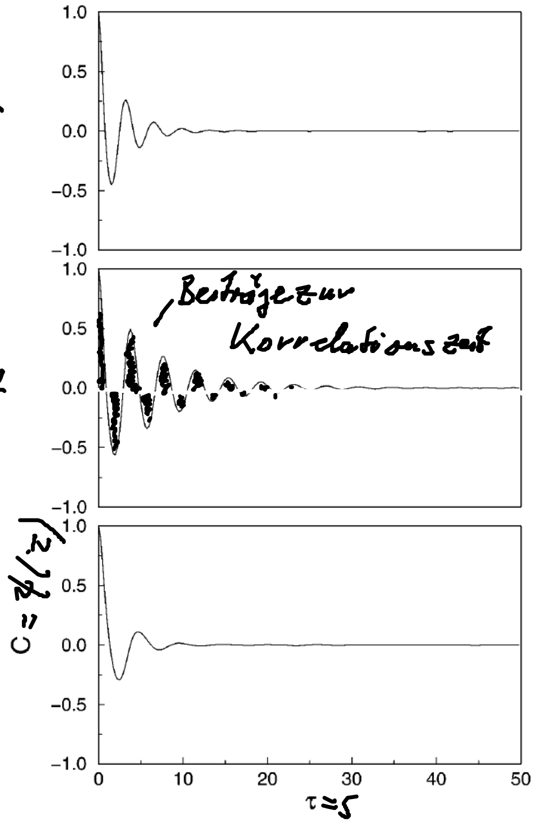
D=0.25

D=0.07

D=0.02

anregbar

FIG. 1. The dynamics of the Fitz Hugh–Nagumo system [Eqs. (1), (2)] for $a = 1.05$, $\varepsilon = 0.01$, and different noise amplitudes: From bottom to top $D = 0.02$, $D = 0.07$, and $D = 0.25$. The mean durations of pulses are 7, 4, and 3.5, respectively. The activation and the excursion times for one pulse are depicted.



rauschdominiert

$t_{cor}(0.07) > t_{cor}(0.02)$

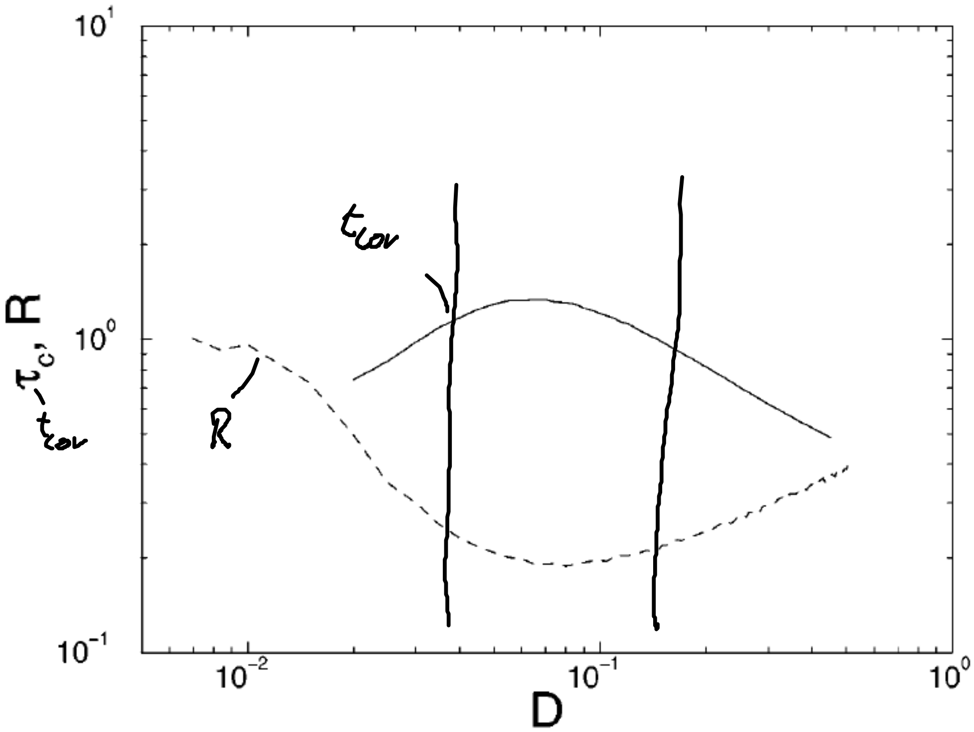
$t_{cor}(0.07) > t_{cor}(0.25)$

optimales Rauschen

zu kleines Rauschen

FIG. 2. The autocorrelation function of the regimes presented in Fig. 1.

Physically, the appearance of coherence resonance is



• maximale Korrelationszeit

• minimale relative Schwankungen vom T_{ISI}

⇒ optimale Rauschstärke

FIG. 3. Correlation time τ_c (solid line) and the noise-to-signal ratio R [Eq. (5), dashed line] vs noise amplitude for the Fitz Hugh–Nagumo system with $a = 1.05$, $\varepsilon = 0.01$.

1. Erwähnung des rauschinduzierten Effekts der Kohärenzresonanz

PHYSICAL REVIEW
LETTERS

VOLUME 71 9 AUGUST 1993 NUMBER 6

Stochastic Resonance without External Periodic Force

Hu Gang

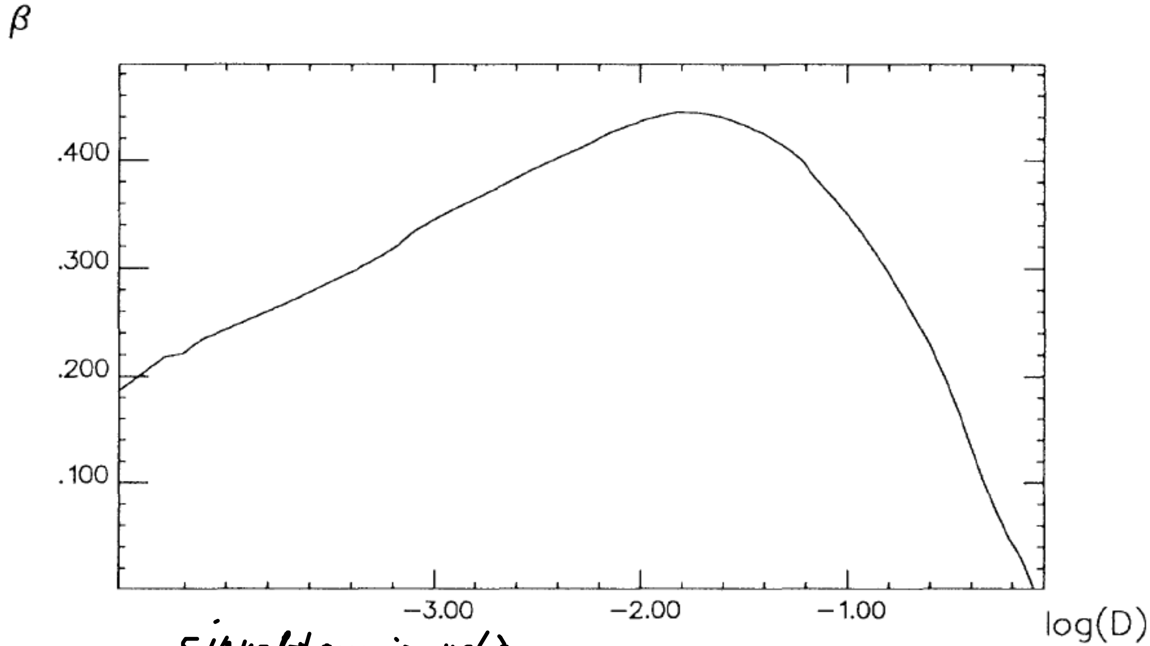
International Centre for Theoretical Physics, Trieste 34100, Italy
and Physics Department, Beijing Normal University, Beijing 100875, People's Republic of China

T. Ditzinger,* C. Z. Ning, and H. Haken

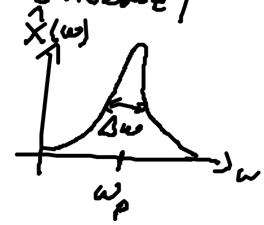
Institut für Theoretische Physik und Synergetik, Universität Stuttgart, Pfaffenwaldring 57/IV, D-7000 Stuttgart 80, Federal Republic of Germany

(Received 21 December 1992)

Kohärenzresonanz am Beispiel des SNIPER-Modell



Signal-Rausch-verhältnis (über Peak im Spektrum der Zeitserie)



signal-to-noise ratio

FIG. 5. The SNR $\beta = h(\Delta\omega/\omega_p)^{-1}$ vs $\log(D)$. A stochastic resonance maximum can be seen.