English summary

3.2 Morris-Lecar model

$$C_m \dot{V} = \underline{I} - g_K\, w(V)\,(V-E_K) - \overbrace{g_{Ca}\, m_\infty(V)\,(V-E_{Ca})}^{\text{fast Ca dynamics}} - g_L\,(V-E_L)$$

$$\tau_w(V)\,\dot{w} = -w + w_\infty(V)$$

with the following functions:

$$m_\infty(V) = \frac{1}{2}\left[1+\tanh\frac{V-V_1}{V_2}\right],\quad w_\infty(V) = \frac{1}{2}\left[1+\tanh\frac{V-V_3}{V_4}\right],\quad \tau_w(V) = \frac{1}{\cosh\frac{V-V_3}{2V_4}}$$

$\Rightarrow$ homoclinic bifurcation

4 Interplay of noise and delay

4.1 Coherence resonance

- constructive role of noise
- coherence resonance: noise-induced oscillations become most regular for finite noise strength
- quantification by the following measures:

(i) correlation time $t_{cor} = \frac{1}{\psi(0)} \int_0^\infty \underset{\substack{\uparrow \\ \text{autocorrelation function}}}{|\psi(s)|}\, ds$

(ii) normalized fluctuations of interspike intervals $R = \frac{\sqrt{\langle T_{ISI}^2\rangle - \langle T_{ISI}\rangle^2}}{\langle T_{ISI}\rangle}$

(iii) signal-to-noise ratio $\beta = \underset{\substack{\uparrow \\ \text{height}}}{h}\, \frac{\omega_{\overset{\text{peak}}{p}}}{\underset{\substack{\uparrow \\ \text{width}\ (h_1 = \frac{h}{\sqrt{e}})}}{\Delta\omega}}$

- example: FitzHugh-Nagumo system

$$\varepsilon \dot{x} = x - \frac{x^3}{3} - y, \quad \dot{y} = x + a + D\xi(t)$$

4.1 Kohärenzresonanz (Fortsetzung)

Grundbegriffe zu stochastischen Prozessen:

- Zeitentwicklung einer Zufallsvariable

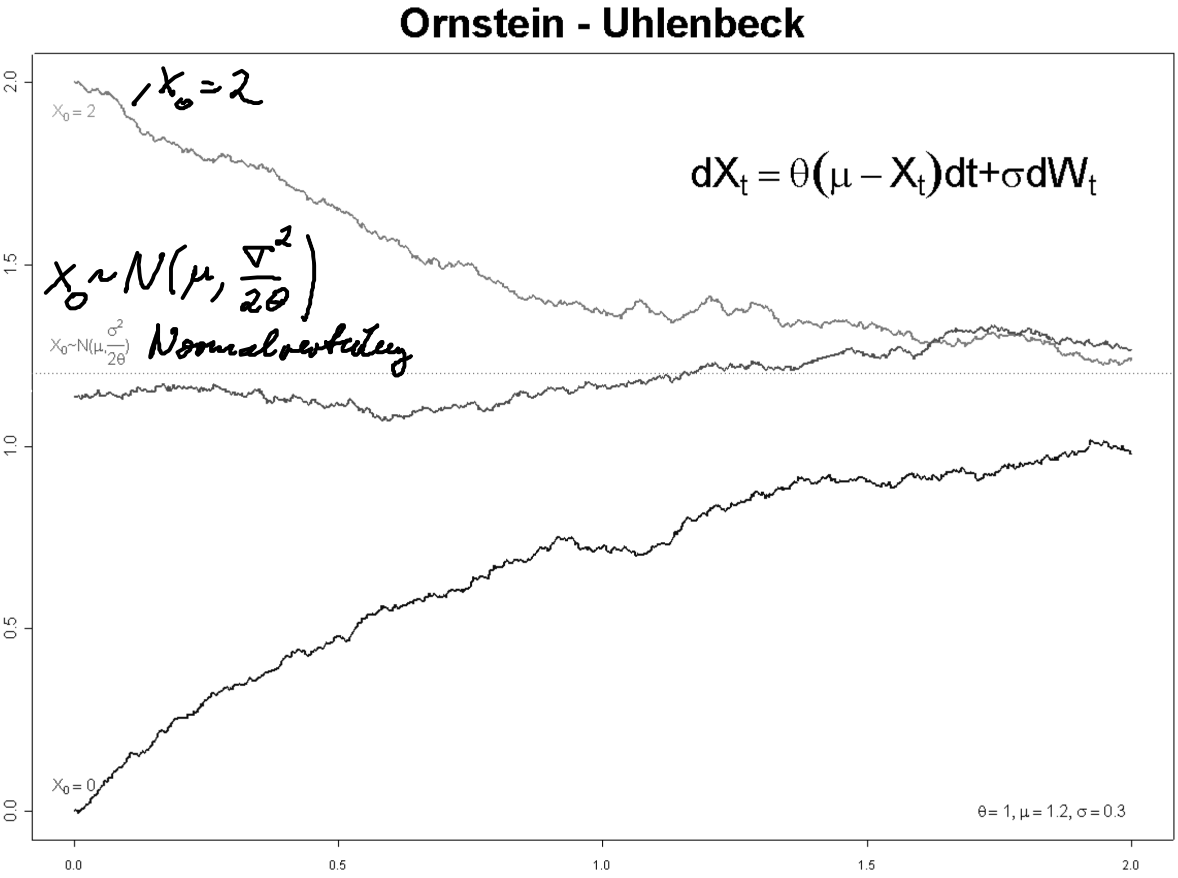
Langevin-Gleichung: fluktuierende, stochastische Kraft $\xi(t)$ (Rauschen)

z.B. Brownsche Bewegung (1827)

$$m\ddot{x} = \underbrace{-\eta}_{\text{Reibung}}\dot{x} + \underbrace{\xi(t)}_{\text{Rauschen}}$$ Bewegungsgleichung eines Teilchens

Rauschen ≙ Kraft durch zufällige Stöße

stochastische Differenzialgleichung: $$\underbrace{\underbrace{dX_t}_{\text{kleine Änderung von } X_t} = \underset{\substack{\uparrow\\ \text{Rück-}\\ \text{stellrate}}}{\Theta}\,(\underset{\substack{\uparrow\\ \text{langfristiger}\\ \text{Mittelwert}}}{\mu} - X_t)\,dt + \overbrace{\underset{\substack{\uparrow\\ \text{Volatilität}}}{\sigma}\,dW_t}^{\text{Rauschen}}}_{\text{Ornstein-Uhlenbeck-Prozess}}$$



, $X_0 = 2$

$X_0 \sim N(\mu, \frac{\sigma^2}{2\theta})$

Normalverteilung

W_t: Zufallsvariable eines Wiener-Prozesses

$\Rightarrow W_0 = 0$

$W_t - W_s \sim N(0, t-s)$

Normalverteilung mit Mittelwert $\mu = 0$ und Varianz $t - s$

zentraler Grenzwertsatz: unkorrelierte Zufallsvariablen gehorchen einer Gauß-Verteilung

Gaußsches weißes Rauschen : $\langle \xi(t) \rangle = 0$ im zeitlichen Mittel keine gerichtete Kraft

$\langle \xi(t)\, \xi(t') \rangle = \delta(t-t')$ unkorreliert, keine Gedächtniseffekte

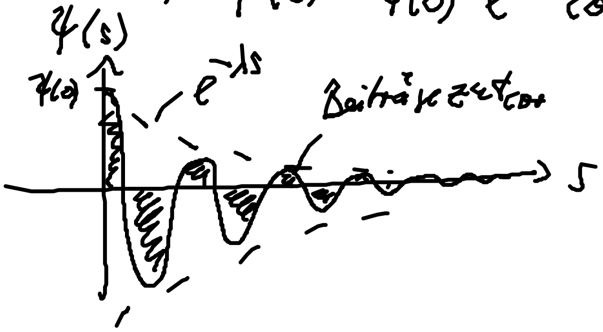
Verteilung der Zufallszahlen $\xi(t)$: Gauß-Verteilung

Autokorrelationsfunktion: $\psi(s) = \Big\langle \big(x(t) - \langle x \rangle\big)\big(\underbrace{x(t+s)}_{\text{um } s \text{ verschobene Zeitserie } x} - \langle x \rangle\big) \Big\rangle$ ← Mittelung über die Zeit

$$\psi(s) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \underset{\uparrow \text{ Annahme } \langle x \rangle = 0}{x(t)}\, x(t+s)\, dt$$ Berechnung über endliches Zeitintervall

Bsp.: linearer stochastischer Prozess : $\dot{x} = -(\lambda + i\omega_0)x + \xi(t)$

$$\Rightarrow \psi(s) = \psi(0)\, e^{-\lambda s} \cos(\omega_0 s)$$



$$t_{cor} = \frac{\psi(0)}{\psi(0)} \int_0^\infty e^{-\lambda s} |\cos(\omega_0 s)|\, ds \approx \frac{2}{\pi \lambda}$$

$$\Rightarrow \psi(s) = \psi(0)\, e^{-\frac{2}{\pi}\frac{s}{t_{cor}}} \cos(\omega_0 s)$$

Korrelationszeit ≙ Abfallrate der Einhüllenden der Autokorrelationsfunktion

spektrale Leistungsdichte : $S(\omega) = \lim_{T \to \infty} \frac{\pi}{T} \big|\underbrace{\hat{x}(\omega, T)}_{= \frac{1}{2\pi}\int_{-T}^{T} e^{i\omega t} x(t)}\big|^2$: Fourier-Transformierte

$$\Rightarrow S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \langle \underset{\uparrow \langle x \rangle = 0}{x(t)}\, x(t+s) \rangle e^{i\omega s}\, ds$$

S ist die Fourier-Transformierte von $\psi(s)$ ≙ Wiener-Khinchin-Theorem

4.2 Zeitverzögerte Rückkopplung

time-delayed feedback control = "Pyragas control"

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Continuous control of chaos by self-controlling feedback

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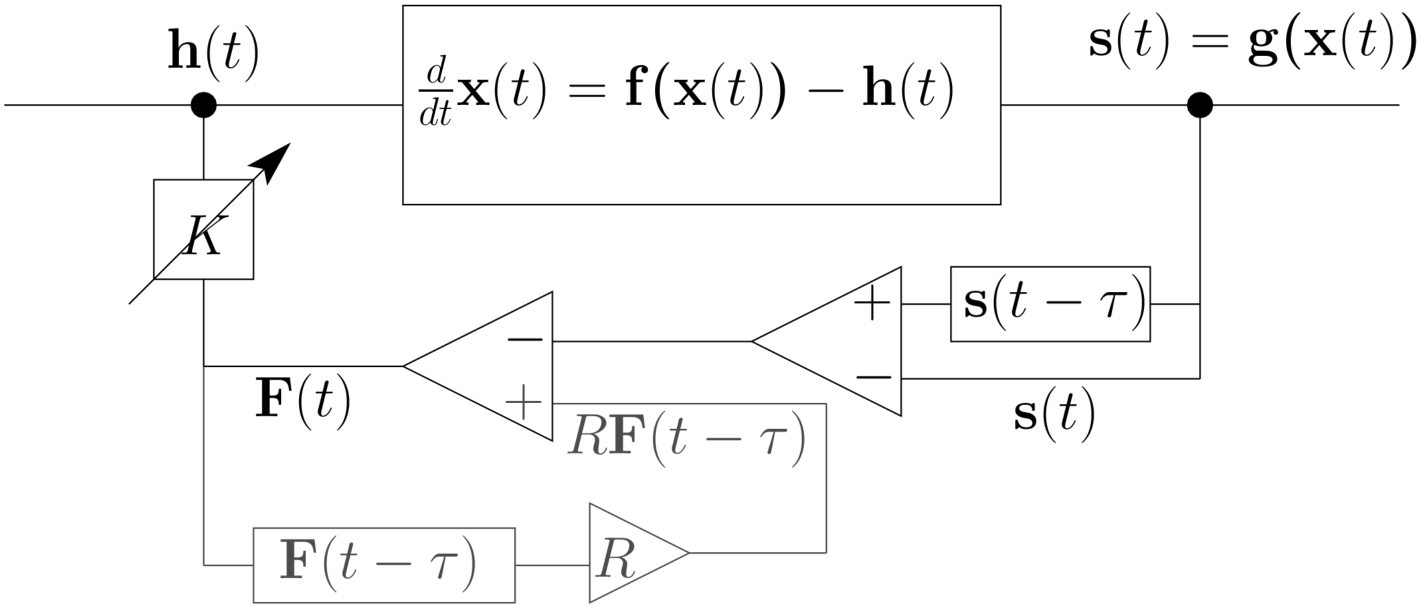
Communicated by J. Flouquet

ursprünglich: Kontrolle instabiler periodischer Orbits/Grenzzyklen in seltsamen Attraktoren chaotischer Systeme

Vgl. SS14, 15, 16 : nichtlineare Dynamik und Kontrolle



Bewegungsgleichung der Form: $\dot{x} = f(x(t)) \underbrace{- K\,[s(t) - s(t-\tau)]}_{\text{zeitverzögerte Rückkopplung}}$

hier: Anwendung auf neuronale Dynamik

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Regular Article

Delay control of coherence resonance in type-I excitable dynamics

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SNIPER

$$\boxed{\dot{x} = x\left(1 - x^2 - y^2\right) + y\,(x - b)} + D\xi + K\left(\underset{x(t-\tau)}{x_\tau} - x\right) \tag{1a}$$

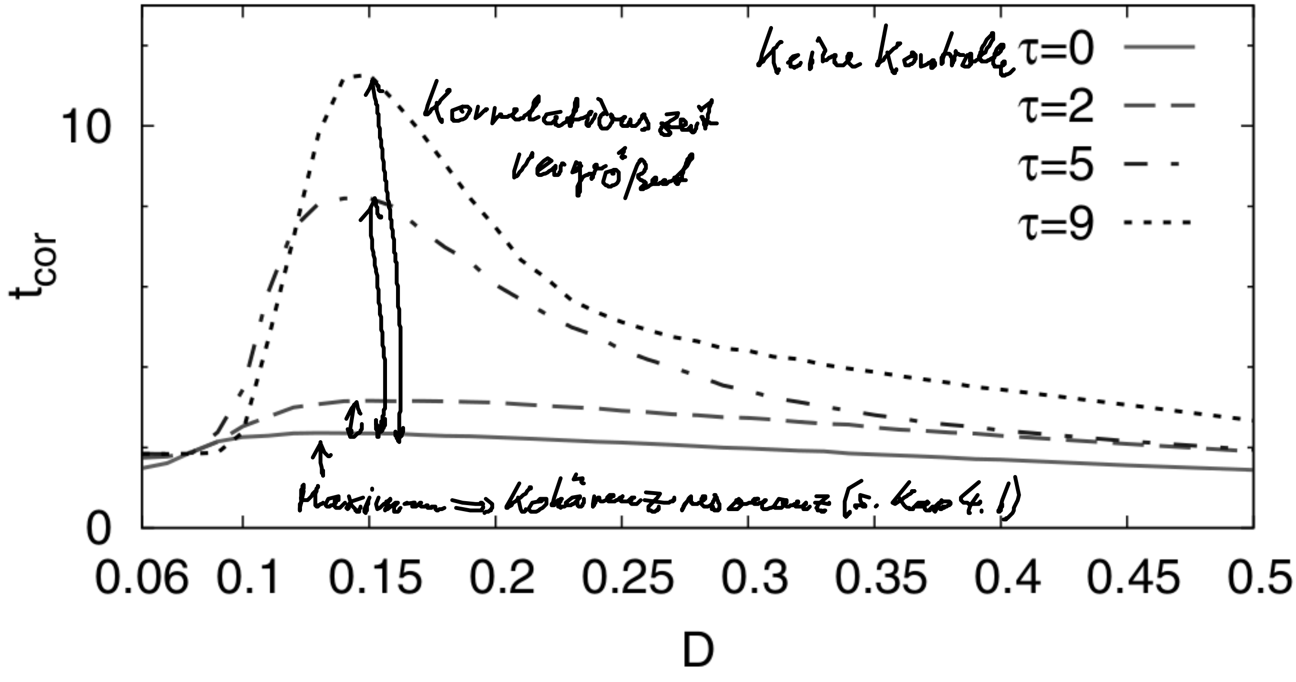
$$\boxed{\dot{y} = y\left(1 - x^2 - y^2\right) - x\,(x - b)} + D\xi + K\left(y_\tau - y\right). \tag{1b}$$

Here x and y are the variables at time t, while x_τ and y_τ denote the respective variables at a delayed time $t - \tau$. The bifurcation parameter $b \in \mathbb{R}$ is a real constant. K denotes the control strength and τ is the delay time. Random input ξ is realized as Gaussian white noise with mean $\langle\xi(t)\rangle = 0$, variance $\langle\xi(t)\xi(t')\rangle = \delta(t - t')$, and noise intensity D. In polar coordinates the system equations for $D = 0$ and $K = 0$ are given by [22]:

$$\dot{r} = r\left(1 - r^2\right) \tag{2a}$$

$$\dot{\varphi} = b - r\cos\varphi. \tag{2b}$$

Ziel: Kontrolle (Verstärkung der Regularität) rauschinduzierter Oszillationen

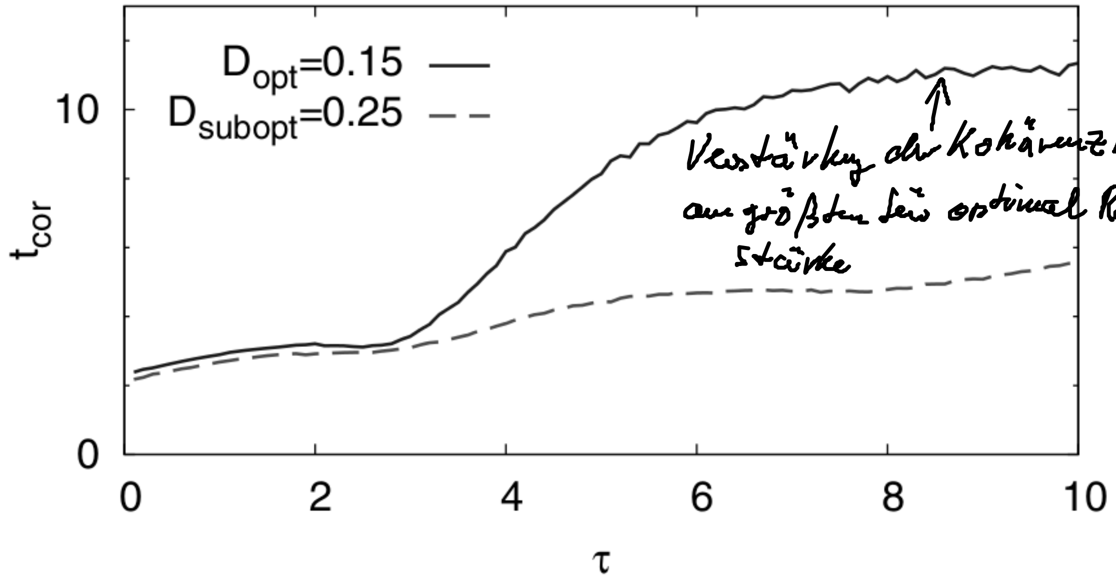


Keine Kontrolle

Korrelationszeit vergrößert

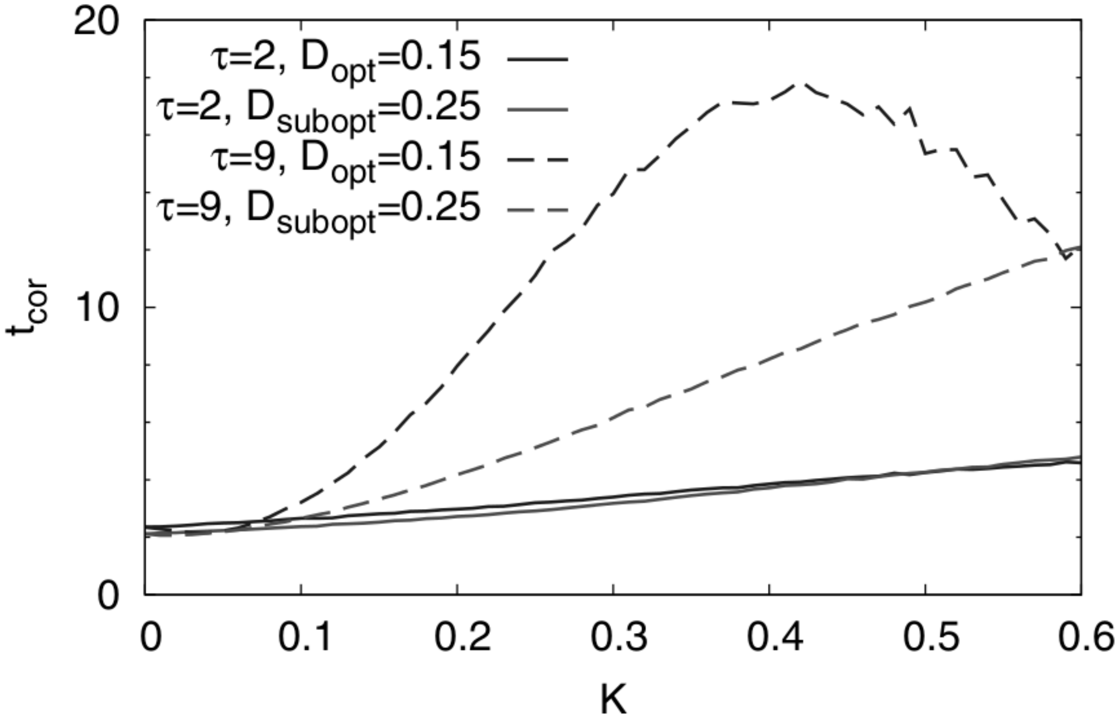
Maximum ⇒ Kohärenzresonanz (s. Kap 4.1)

Correlation time in dependence on the noise intensity D for different time delays τ. The solid (green) curve corresponds to the uncontrolled system ($\tau = 0$). The dashed (red), delays τ. The solid (green) curve corresponds to the uncontrolled system ($\tau = 0$). The dashed (red), dash-dotted (blue), and dotted (black) curves refer to values of $\tau = 2, 5$, and 9, respectively. Other parameters: $b = 0.95$ and $K = 0.25$.



Verstärkung der Kohärenzresonanz am größten bei optimaler Rausch-stärke

Correlation time t_{cor} in dependence on the time delay τ for two values of the noise intensity D. The dashed (red) curve corresponds to $D_{subopt} = 0.25$ and the solid (blue) curve refers to $D_{opt} = 0.15$. Other parameters: $b = 0.95$ and $K = 0.25$.



Correlation time t_{cor} in dependence on the control strength K for two values of the noise intensity D and two values of the delay time τ. The gray (red) and black (blue) curves depict the cases of D_{subopt} and D_{opt}, respectively. The solid and dashed lines correspond to $\tau = 2$ and $\tau = 9$, respectively. Other parameter: $b = 0.95$.