

# English summary

## 5.4 Dynamics on empirical networks

• Kuramoto phase oscillators coupled via networks extracted from neuroimaging data

$$\dot{\theta}_i = \omega_i + c \sum_{j=1}^N a_{ij} \sin(\theta_j - \theta_i - \alpha) \quad i=1, \dots, N$$

intrinsic frequency

adjacency matrix

phase frustration parameter

$\{a_{ij}\}_{i,j=1, \dots, N}$ : obtained from functional [fMRI] and/or structural/anatomical [DTI] measurements

workflow:

- generate  $\{a_{ij}\}$
- run simulations  $\rightarrow \theta_i(t)$
- neuronal activities via  $V_i(t) = \sin \theta_i(t)$
- BOLD signal (blood response) via Balloon-windkessel model
- calculate functional correlation from simulated BOLD signal
- compare with empirical BOLD signal

observations: - best agreement for some  $\alpha$ -values and network densities (threshold of  $\{a_{ij}\}$ )

- remote synchronization of indirectly coupled brain regions/nodes
- level of synchrony with fluctuations

quantified by order parameter  $R(t) = \frac{1}{N} \left| \sum_{j=1}^N e^{i\theta_j(t)} \right|$

## 6 Synchronization in Networks

### 6.1 Synchronization and Inhibition (vgl. 5.3)

FitzHugh-Nagumo (type II):

Hopf bifurcation

$$\begin{aligned} \epsilon \dot{u} &= u - \frac{u^3}{3} - v \\ \dot{v} &= u + a \end{aligned}$$

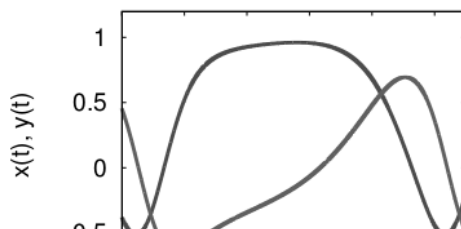
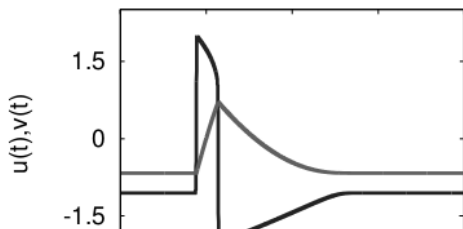
a: threshold parameter  
 $\epsilon$ : time-scale parameter

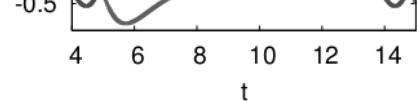
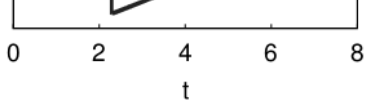
SNIPER (type I):

Saddle-node bifurcation

$$\begin{cases} \dot{r} = r(1-r^2) \\ \dot{\varphi} = b(1-r \cos \varphi) \\ \dot{x} = x(1-x^2-y^2) + y(x-b) \\ \dot{y} = x(1-x^2-y^2) - x(x-b) \end{cases}$$

b: bifurcation parameter





Frage: Stabilität von Synchronisation in einem Netzwerk?

=> master stability function *↖* Substrukt zur synchronen Lösung

Zusammengefasst: Berechne transversale Lyapunov-Exponenten in Abhängigkeit der Eigenwerte der Kopplungsmatrix

Test: alle Exponenten (für gegebenes Kopplungsmatrix)  $< 0 \Rightarrow$  stabil  
mind. 1 Exponent (für gegebenes Kopplungsmatrix)  $> 0 \Rightarrow$  instabil

Coupled system:

z.B.:  $\underline{x}_i = \begin{pmatrix} u_i \\ v_i \end{pmatrix}$  rechte Seite

$$\dot{\underline{x}}_i = \underbrace{F(\underline{x}_i)}_{\text{lokale Dynamik}} + C \sum_{j=1}^N \underbrace{G_{ij}}_{\text{verallgemeinerte Adjazenzmatrix } G_{ij} \in \mathbb{R}} \underbrace{H[\underline{x}_j(t-\tau) - \underline{x}_i(t)]}_{\substack{\text{welche Komponente von } \underline{x}_j \text{ koppelt in welche} \\ \text{Komponente von } \underline{x}_i}}$$

G: coupling matrix  
H: coupling scheme

Constant row sum of G:  $\sum_{j=1}^N G_{ij} =: g$  here:  $g = 1$  WLOG

damit Kopplungsterm im synchronen Fall für alle  $\underline{x}_i$  gleich ist

Synchronization manifold:  $\underline{x}_1 = \underline{x}_2 = \dots = \underline{x}_N =: \underline{x}_s$

$$\dot{\underline{x}}_s = F(\underline{x}_s) + CH[\underline{x}_s(t-\tau) - \underline{x}_s(t)]$$

Spezialfall: Reduktion von N Elementen auf 1 Element

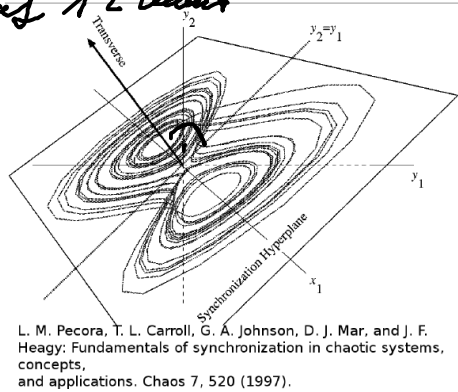
Is the synchronization manifold stable?

=> Largest transversal Lyapunov exponent  $\Lambda$

=> Master stability function (MSF)

(Diagonalize the system such that the dynamics in the different directions of the phasespace are separated => N 2-dim equations)

im FitzHugh-Nagumo, SNIPER...



L. M. Pecora, T. L. Carroll, G. A. Johnson, D. J. Mar, and J. F. Heagy: Fundamentals of synchronization in chaotic systems, concepts, and applications. Chaos 7, 520 (1997).

Combine the dynamics of N nodes in one equation:

$$\dot{\underline{x}} = \mathbb{1} \otimes \underbrace{F(\underline{x})}_{\text{local}} + C \underbrace{G}_{\text{topology}} \otimes \underbrace{H}_{\text{coupling}} \underline{x}_\tau$$

$\underline{x} = (x_1, x_2, \dots, x_N)$

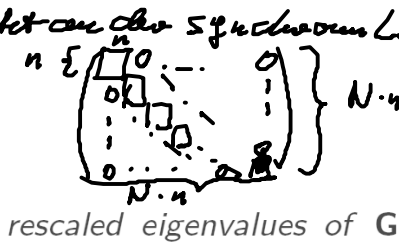
Variational equation:  $\delta \underline{x} = \begin{pmatrix} \delta x_1 \\ \delta x_2 \\ \vdots \\ \delta x_N \end{pmatrix}$  mit  $\delta x_1 = \underline{x}_1, -\underline{x}_s$

$$\delta \dot{\underline{x}} = \mathbb{1} \otimes \underline{DF} \delta \underline{x} + C \underline{G} \otimes \underline{H} \delta \underline{x}_\tau$$

Jacobi-Matrix ausgehend von der Synchronen Lösung

Block diagonalize:

⇒ Master stability equation



für jeden Eigenwert von  $\underline{G}$   $n$ -fache  $k$ -koppertes  
 $N$  unkoppelte Gleichungen durch Blockdiagonalisierung

$$\delta \dot{x}_j = DF \delta x_j + \underbrace{(\alpha_j + i\beta_j)}_{\text{rescaled eigenvalues of } G} H \delta x_j$$

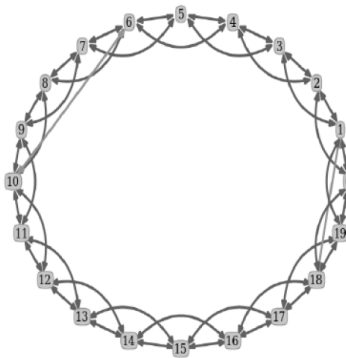
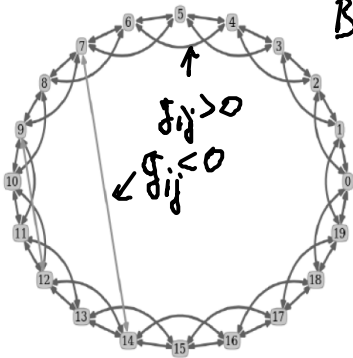
Lyapunov exponents as a function of the rescaled eigenvalues (topology!)

Berechne Lyapunov-Exponenten in Abhängigkeit von  $\alpha$  und  $\beta$  (für alle Kopplungsmatrizen)

Two realizations with additional inhibition ( $N = 20, k = 2, p = 0.05$ )

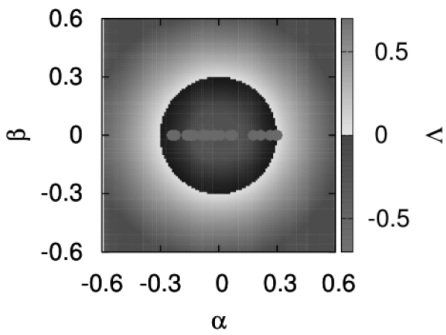
Bsp. aus Kap. 5.3

Fiktive Netzwerke mit positiven und negativen Einträgen  $g_{ij}$

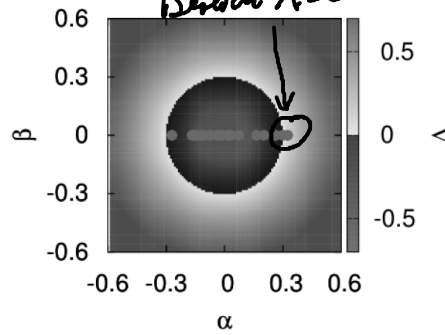


- Vorteil: lineare Berechnung von  $\Lambda(\alpha, \beta)$
- Anschließend Einsetzen der Eigenwerte von  $\underline{G}$  zum Test  $\Lambda(\alpha_j, \beta_j) \geq 0$   $j = 1, \dots, N$
- Symmetrisches  $\underline{G} \Rightarrow \beta_1 = \dots = \beta_N = 0$

alle Eigenwerte im Bereich  $\Lambda < 0$



Eigenwert im Bereich  $\Lambda > 0$



Synchronization

Desynchronization

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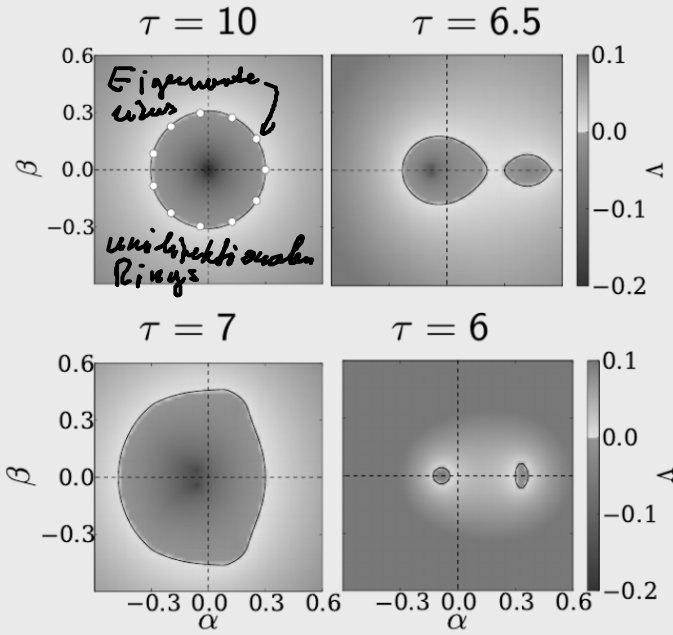
Regular Article

# Synchronisation in networks of delay-coupled type-I excitable systems

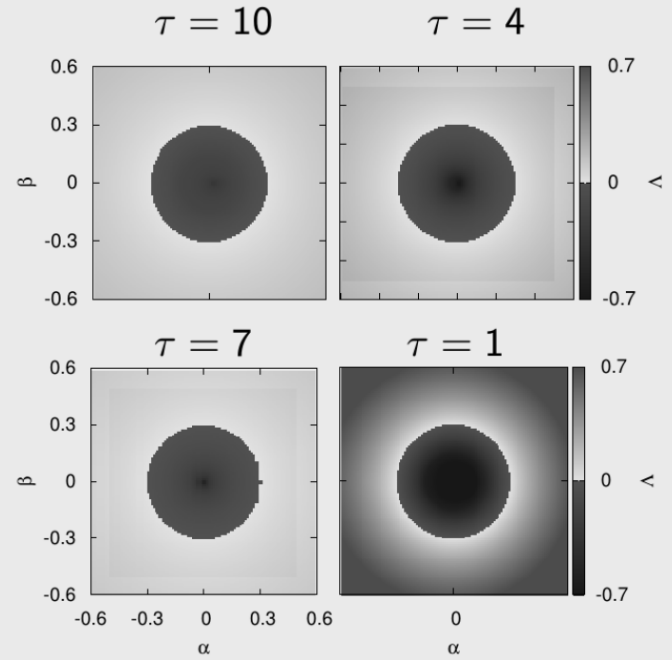
⇒ SNIPER

A. Keane<sup>1</sup>, T. Dahms<sup>1</sup>, J. Lehnert<sup>1</sup>, S.A. Suryanarayana<sup>1,2</sup>, P. Hövel<sup>1,3,4</sup>, and E. Schöll<sup>1,a</sup>

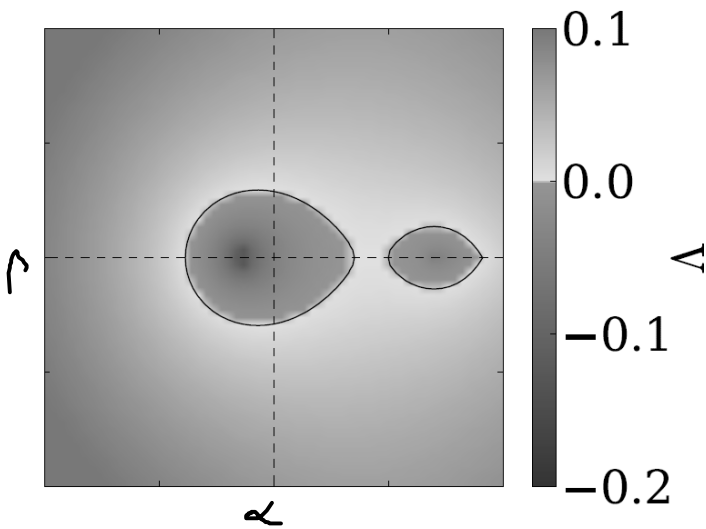
SNIPER:  $b=0.95$



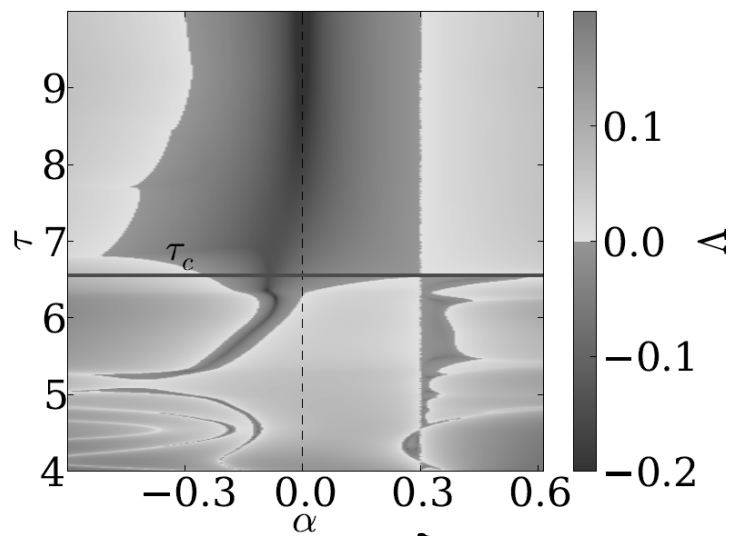
FHN:  $a=1.3, \epsilon=0.01$



Circular stability region for SNIPER and FHN in the case of large delay



$\tau = 6.5$



Symmetrische Kopplungsmatrizen  $\beta=0$

$\Rightarrow$  Stability islands for small delay

100 SNIPER - Elemente auf einem Ring mit  $k$  Nachbarn und zufällige inhibitorische Links hinzugefügt mit Wahrscheinlichkeit  $p$

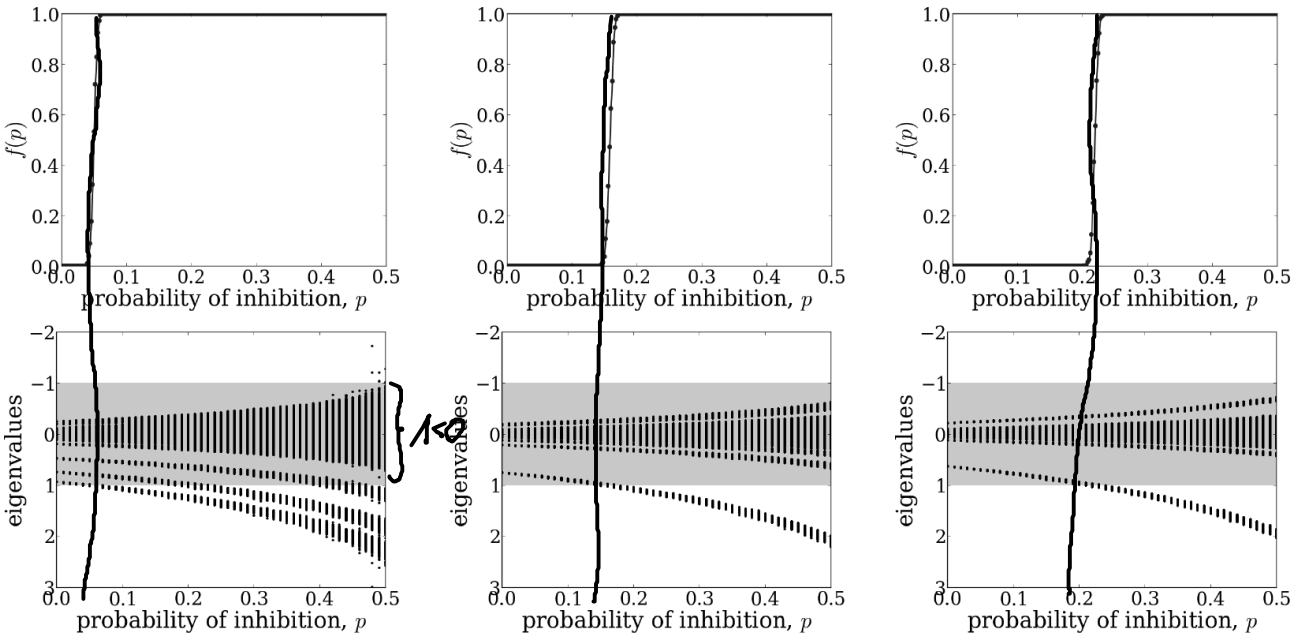
$k = 20$

$k = 40$

$k = 50$

$\tau = 10$

Wahrscheinlichkeit für Desynchronisation = 100%



$k$  number of connected nearest neighbors (underlying regular network)

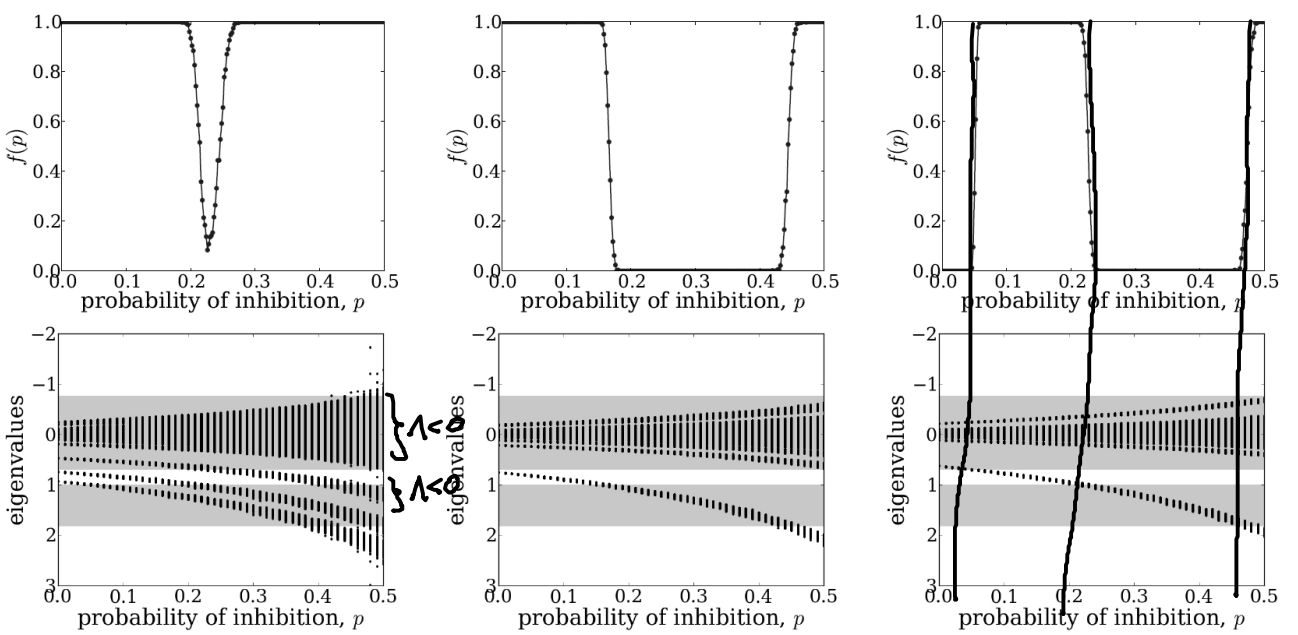
$k = 20$

$k = 40$

$k = 50$

$\tau = 6.5$

mehrfache Übergänge von stabilen zu instabilen Synchronisationsmöglichkeiten



$k$  number of connected nearest neighbors (underlying regular network)