English summary

3. Physiological models

3.1 Hodgkin-Huxley model

first principles: physical, electrical, and chemical fundamentals such as Ohm's law, Kirchhoff's junction rule, Nernst potential

$$c_m \dot{V} = -\bar{g}_K n^4 (V - E_K) - \bar{g}_{Na} m^3 h (V - E_{Na}) - \bar{g}_L (V - E_L)$$

E_K, E_{Na}, E_L: reversal potentials of K, Na, L

V: membrane potential

$\bar{g}_K, \bar{g}_{Na}, \bar{g}_L$: (maximum) conductances of K, Na, leak channels

$n(V)$: gating variable (K activation) $\quad \dot{n} = \alpha_n(V)(1-n) - \beta_n(V)\, n$

$m(V)$: " (Na activation) $\quad \dot{m} = \alpha_m(V)(1-m) - \beta_m(V)\, m$

$h(V)$: " (Na inactivation) $\quad \dot{h} = \alpha_h(V)(1-h) - \beta_h(V)\, h$

or equivalently:

$$\tau_n(V)\, \dot{n} = -n + n_\infty(V)$$

$$\tau_m(V)\, \dot{m} = -m + m_\infty(V)$$

$$\tau_h(V)\, \dot{h} = -h + h_\infty(V)$$

$$\alpha_n(V) = 0.01 \frac{10 - V}{e^{(10-V)/10} - 1}$$

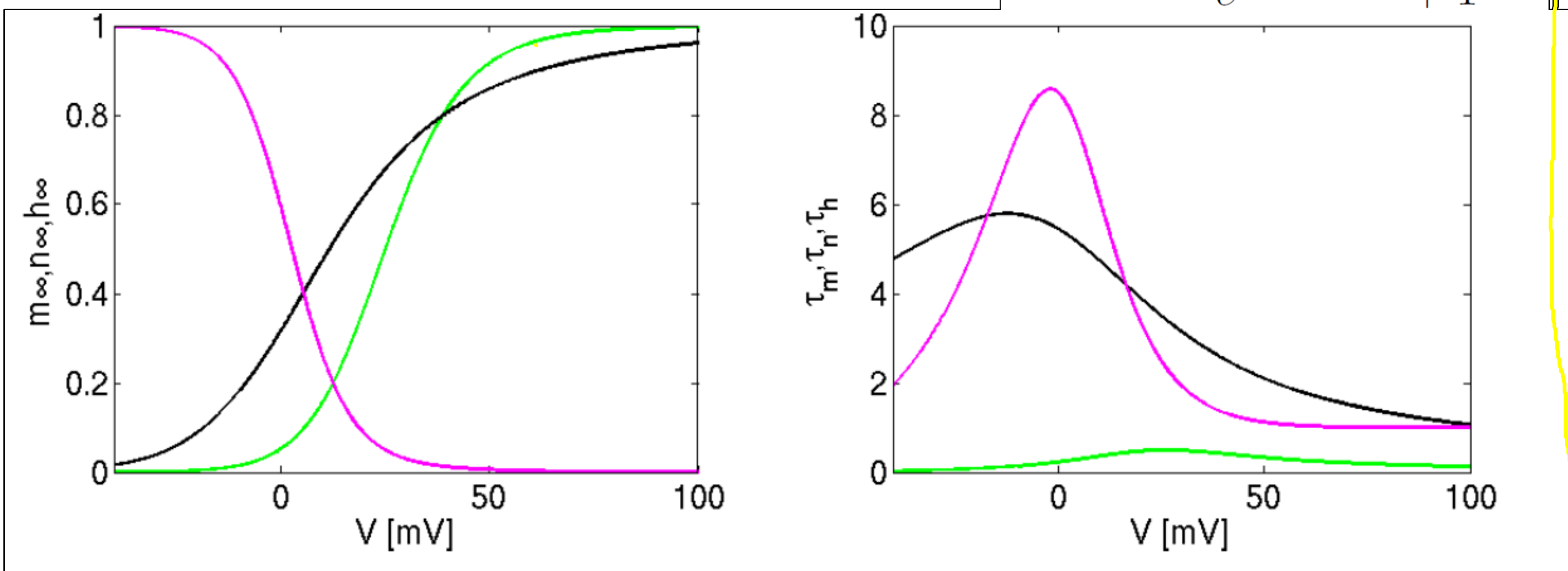
$$\beta_n(V) = 0.125 e^{-V/80}.$$

$$\alpha_m(V) = 0.1 \frac{25 - V}{e^{(25-V)/10} - 1}$$

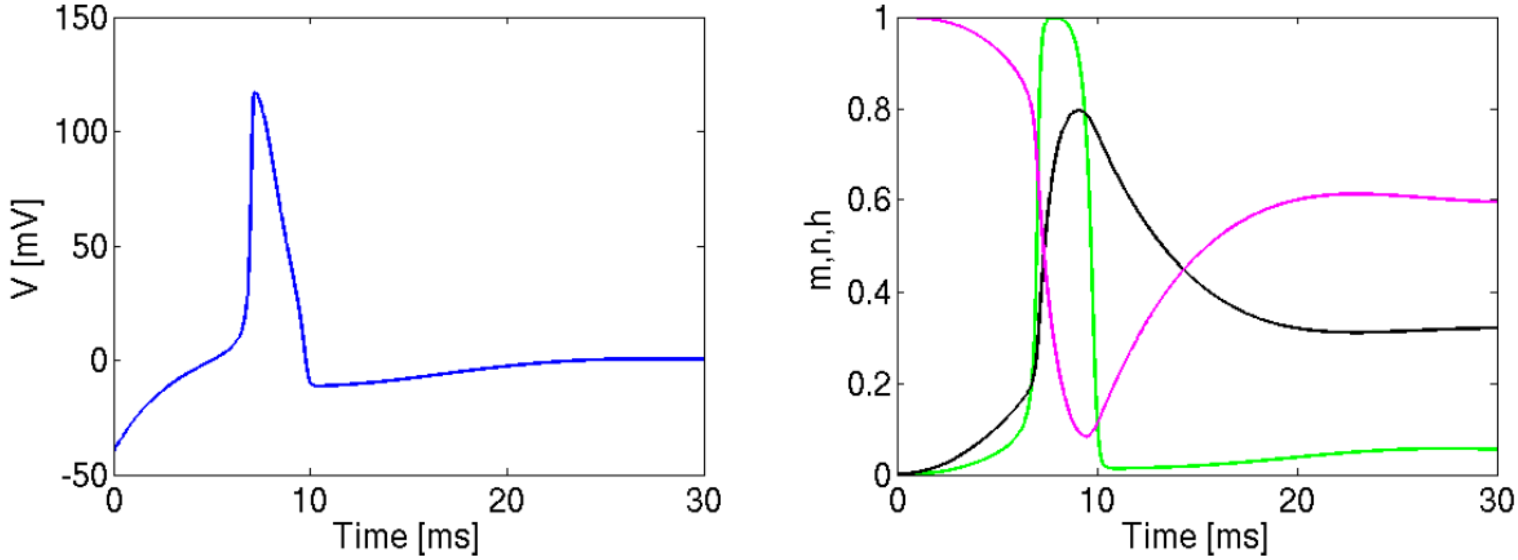
$$\beta_m(V) = 4 e^{-V/18}$$

$$\alpha_h(V) = 0.07 e^{-V/20}$$

$$\beta_h(V) = \frac{1}{e^{(30-V)/10} + 1}.$$



Left panel:Steady state activation and inactivaion functions. Right panel: Voltage dependent time constants, for sodium activation m (green) and inactivation h (pink) and potassium activation n (black).



Left panel: Membrane voltage. Right panel: Activation potential, for sodium activation m (green) and inactivation h (pink) and potassium activation n (black).

3.2 Morris-Lecar model

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VOLTAGE OSCILLATIONS IN THE BARNACLE GIANT MUSCLE FIBER

CATHERINE MORRIS AND HAROLD LECAR, *Laboratory of Biophysics, National Institute of Neurological and Communicative Disorders and Stroke, National Institutes of Health, Bethesda, Maryland 20205*

2-Variablenmodell:

$$C_m \dot{V} = \underset{\uparrow}{I} - g_L(V-E_L) - \underbrace{g_{Ca}\, m_\infty(V)(V-E_{Ca})}_{} - g_K\, w(V)(V-E_K)$$

Bifurkationsparameter
zusätzlicher Strom

Calcium immer im Gleichgewicht (Fixpunkt) aufgrund schneller Dynamik

$\tau_m \dot{m} = -m + m_\infty(V)$

$\dot{m} \overset{!}{=} 0 \Rightarrow m = m_\infty(V)$

$$\tau_w(V)\, \dot{w} = -w + w_\infty(V)$$

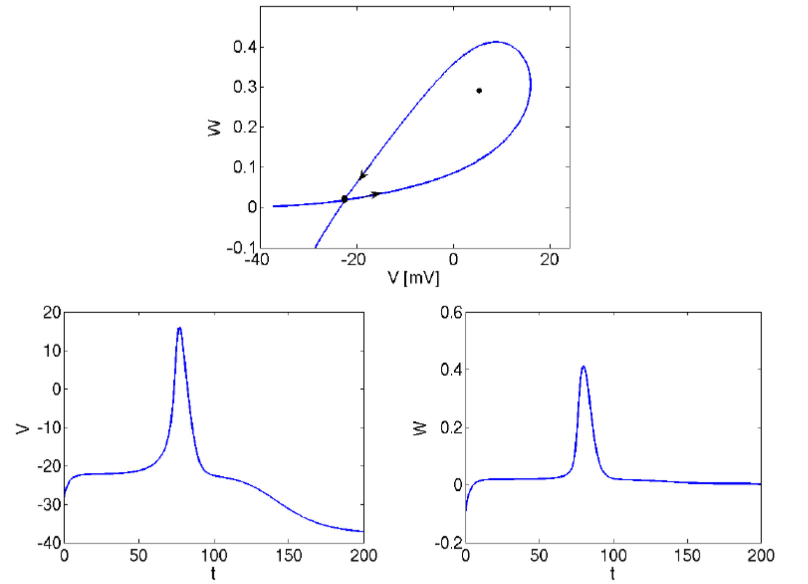
Parameter (Potenzial abhängig):

$$m_\infty(V) = \frac{1}{2}\left[1 + \tanh \frac{V - V_1}{V_2}\right] = \left[1 + \exp\left(-2\frac{V - V_1}{V_2}\right)\right]^{-1}$$

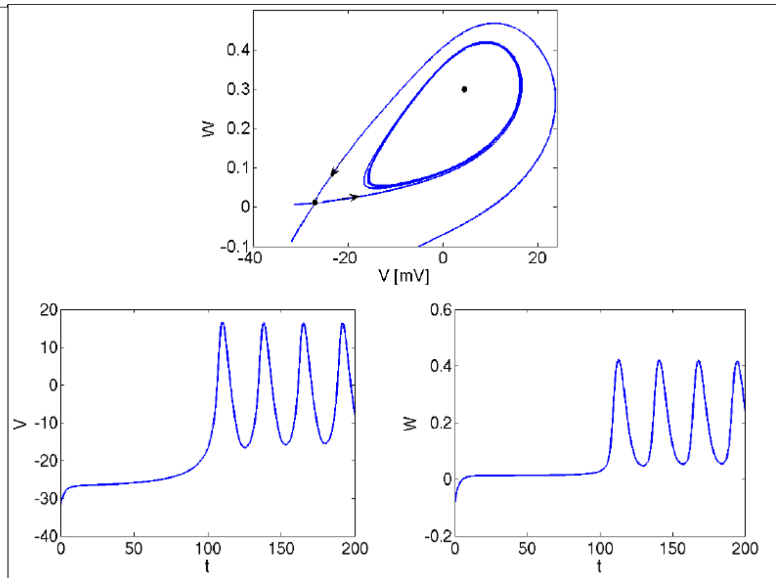
$$w_\infty(V) = \frac{1}{2}\left[1 + \tanh \frac{V - V_3}{V_4}\right] = \left[1 + \exp\left(-2\frac{V - V_3}{V_4}\right)\right]^{-1}$$

$$\tau_w(V) = \frac{1}{\cosh \frac{V - V_3}{2V_4}}$$

V_1, V_2, V_3, V_4: Parameter zum Einstellen des Fixpunkts



The phase portrait of the homoclinic bifurcation present in the Morris-Lecar model at $I_0 = 35$ and corresponding time series for v and ω.



The phase portrait of Morris-Lecar at $I_0 = 39.5$ (oscillatory regime) and corresponding time series for v and ω.

Dynamik: homokline Bifurkation (s. Kap 1)

4. Wechselspiel von Rauschen und Zeitverzögerung

bisher: deterministische dynamische Systeme, jetzt: stochastische dynamische Systeme

4.1 Kohärenzresonanz

4.2 zeitverzögerte Rückkopplung

Zeitverzögerungen in gekoppelten Systemen in Kapiteln 5 und 6

4.1 Kohärenzresonanz

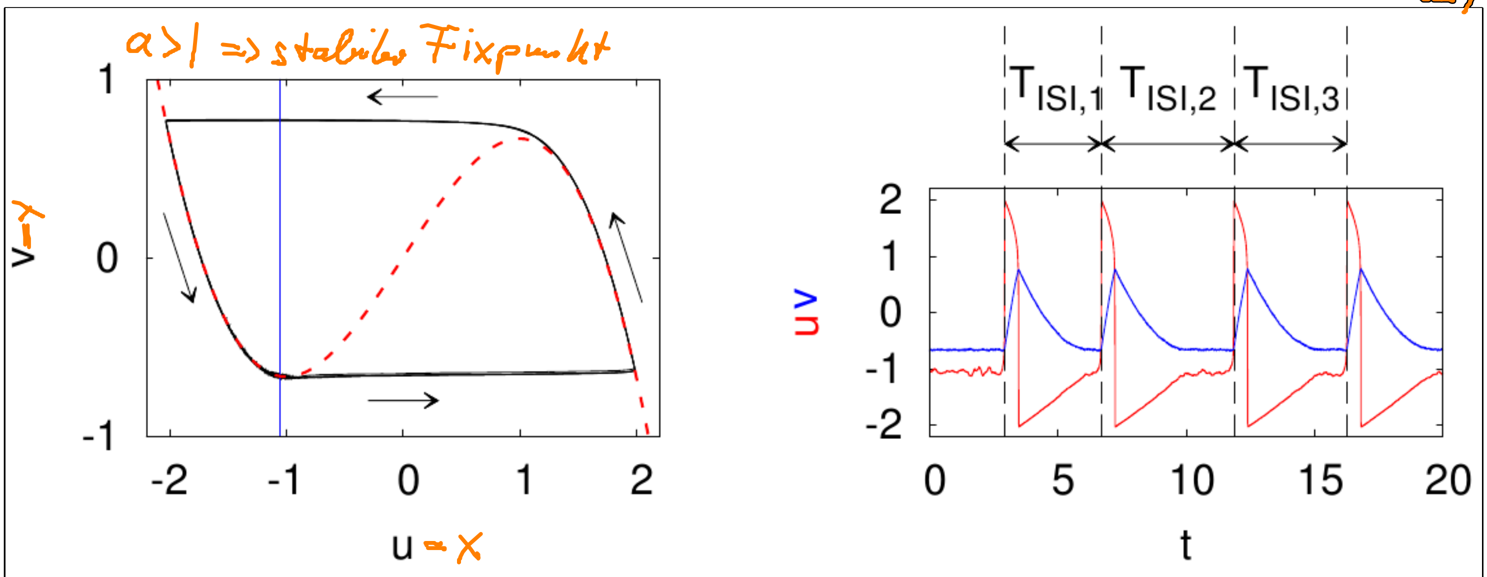
- konstruktiver Einfluss von Rauschen
  $\Rightarrow$ Regularität rauschinduzierter Oszillationen (optimale Rauschstärke)

Bsp.: FitzHugh-Nagumo-System mit Rauschen

$$\varepsilon \dot{x} = x - \frac{x^3}{3} - y$$

$$\dot{y} = x + a \underbrace{+ D\xi(t)}_{\text{additives Rauschen}}$$

rauschinduzierte Oszillationen (Schwankungen in der Periode T_{ISI})

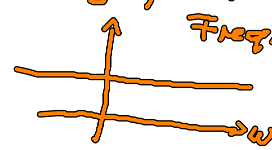


$\xi(t)$: Gauß'sches weißes Rauschen

spektrale Leistungsdichte: $S(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \langle \xi(t)\xi(t+s)\rangle e^{i\omega s}\, ds = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(s)\, e^{i\omega s}\, ds = \frac{1}{2\pi}$

$\langle \xi(t)\xi(t')\rangle = \delta(t-t')$ unkorreliert

keine bevorzugte Frequenz => weiß



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Coherence Resonance in a Noise-Driven Excitable System

Arkady S. Pikovsky* and Jürgen Kurths*

Max–Planck–Arbeitsgruppe "Nichtlineare Dynamik" an der Universität Potsdam Am Neuen Palais 19, PF 601553, D-14415, Potsdam, Germany

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We study the dynamics of the excitable Fitz Hugh–Nagumo system under external noisy driving. Noise activates the system producing a sequence of pulses. The coherence of these noise-induced oscillations is shown to be maximal for a certain noise amplitude. This new effect of coherence resonance is explained by different noise dependencies of the activation and the excursion times. A simple one-dimensional model based on the Langevin dynamics is proposed for the quantitative description of this phenomenon. [S0031-9007(97)02349-1]

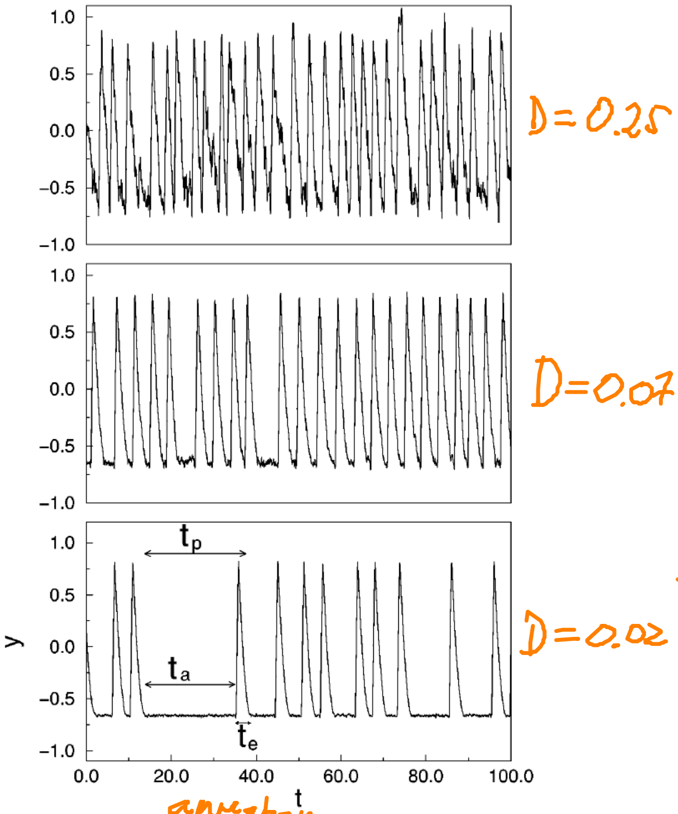
Maße zum Quantifizieren der Regularität:

• Korrelationszeit: $t_{cor} = \frac{1}{\psi(0)} \int_0^\infty |\psi(s)|\, ds$, $\psi(0) = \langle [x(t)-\langle x\rangle][x(t)-\langle x\rangle]\rangle = \langle [x(t)-\langle x\rangle]^2\rangle$ Varianz

Autokorrelationsfunktion: $\psi(s) = \langle [x(t)-\langle x\rangle][x(t-s)-\langle x\rangle]\rangle$

$\langle x\rangle = 0$: $= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} dt\, x(t)\, x(t+s)$

• normierte Fluktuationen der Interspikeintervalle: $R = \frac{\sqrt{\langle T_{ISI}^2\rangle - \langle T_{ISI}\rangle^2}}{\langle T_{ISI}\rangle}$



anregbar

FIG. 1. The dynamics of the Fitz Hugh–Nagumo system [Eqs. (1), (2)] for $a = 1.05$, $\varepsilon = 0.01$, and different noise amplitudes: From bottom to top $D = 0.02$, $D = 0.07$, and $D = 0.25$. The mean durations of pulses are 7, 4, and 3.5, respectively. The activation and the excursion times for one pulse are depicted.

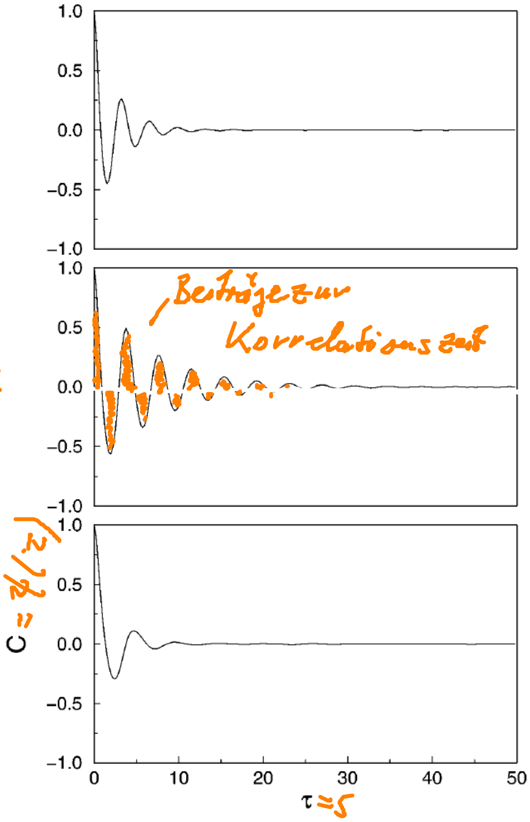


FIG. 2. The autocorrelation function of the regimes presented in Fig. 1.

rauschdominiert

$t_{cor}(0.07) > t_{cor}(0.02)$

$t_{cor}(0.07) > t_{cor}(0.25)$

optimales Rauschen

zu kleines Rauschen

Physically, the appearance of coherence resonance is

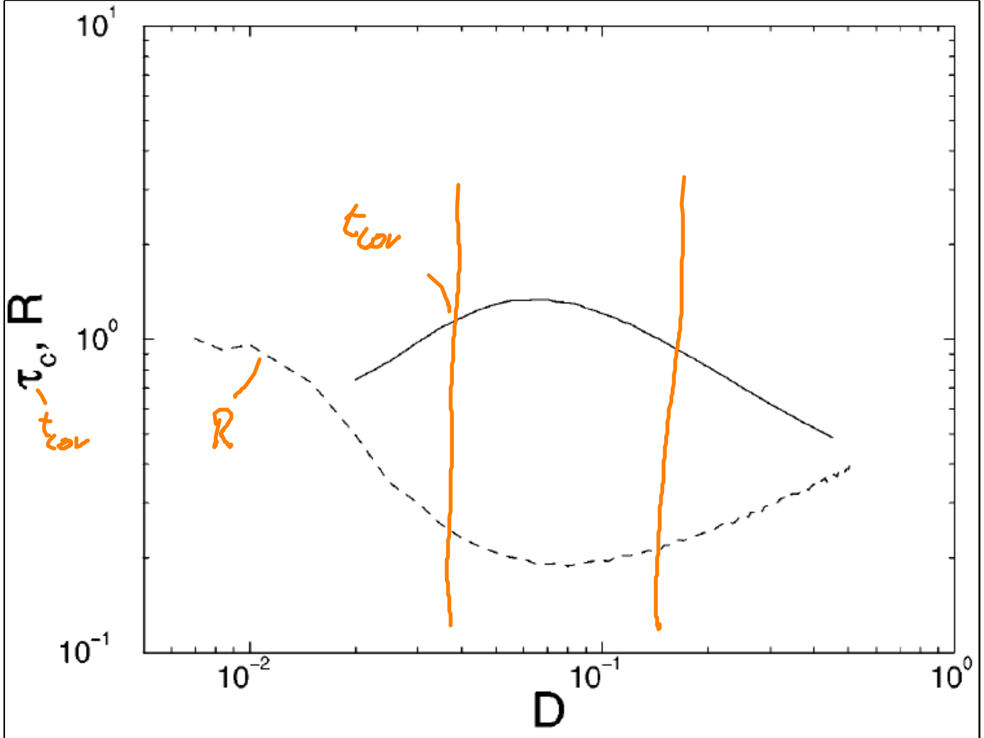


FIG. 3. Correlation time τ_c (solid line) and the noise-to-signal ratio R [Eq. (5), dashed line] vs noise amplitude for the Fitz Hugh–Nagumo system with $a = 1.05$, $\varepsilon = 0.01$.

- maximale Korrelationszeit
- minimale relative Schwankungen von $T_{\underline{ISI}}$

⇒ optimale Rauschstärke

1. Erwähnung des rauschinduzierten Effekts der Kohärenzresonanz

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Stochastic Resonance without External Periodic Force

Hu Gang
International Centre for Theoretical Physics, Trieste 34100, Italy and Physics Department, Beijing Normal University, Beijing 100875, People's Republic of China

T. Ditzinger,* C. Z. Ning, and H. Haken
Institut für Theoretische Physik und Synergetik, Universität Stuttgart, Pfaffenwaldring 57/IV, D-7000 Stuttgart 80, Federal Republic of Germany

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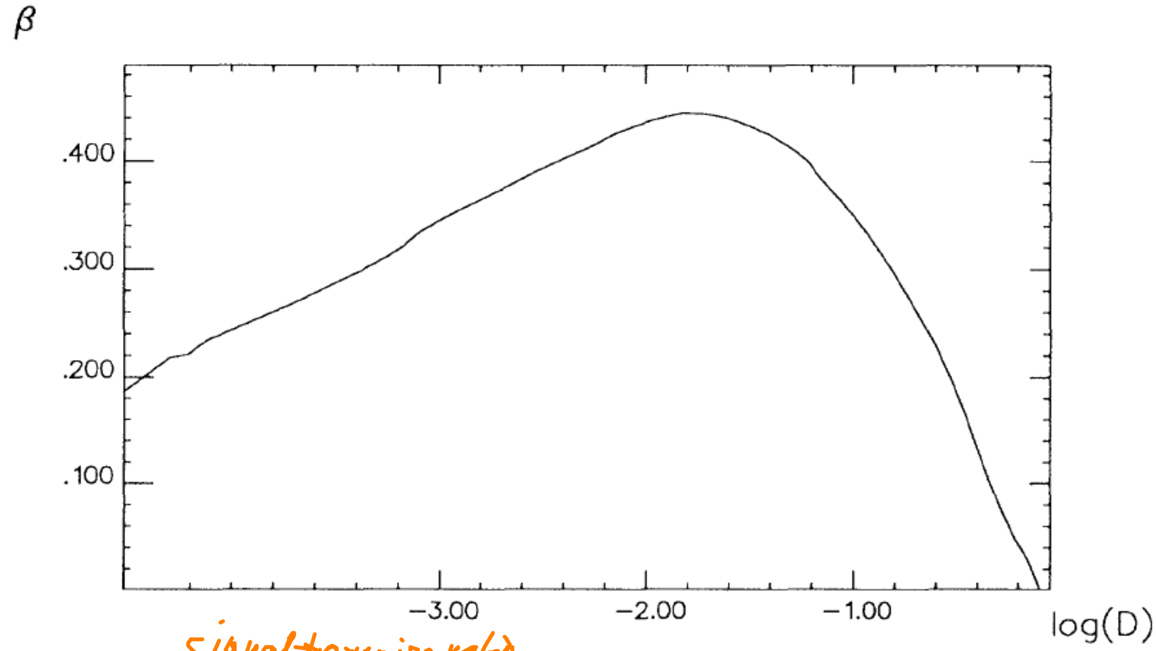
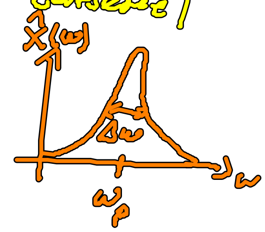
Kohärenzresonanz am Beispiel des SNIPER-Modell



Signal-Rausch-verhältnis (über Peak im Spektrum der Zeitserie)



signal to noise ratio

FIG. 5. The SNR $\beta = h(\Delta\omega/\omega_p)^{-1}$ vs $\log(D)$. A stochastic resonance maximum can be seen.