NON-MARKOVIAN QUANTUM FEEDBACK CONTROL OF PHOTON STATISTICS AND QUANTUM MANY BODY DYNAMICS

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*in collaboration with
N. Nemet, L. Droenner, M. Strauß, S. Reitzenstein, S. Parkins, M. Heyl, and A. Knorr
• Non-Markovian signatures in Quantum Optics: Wigner delay
• Bypassing non-Markovian decoherence via quantum feedback
• Selective photon-probability control in the two-photon regime
• Stabilizing a discrete time crystal against dissipation
• **Non-Markovian signatures in Quantum Optics: Wigner delay**
  • Bypassing non-Markovian decoherence via quantum feedback
  • Selective photon-probability control in the two-photon regime
  • Stabilizing a discrete time crystal against dissipation

*with Max Strauß, Stephan Reitzenstein*
Non-Markovian signatures in Wigner delays

Wigner delay occurs between absorption and emission processes of a single quantum dot

 Strauß, AC et al, PRL 122, 107401 (2019); arXiv: 1805.06357v1
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Wigner delay also strongly dependent on the excitation power if not in the Heitler regime

\[ \Omega(t) = \Omega_L \sqrt{\frac{\pi}{(2\tau^2)}} \exp \left[ -\frac{(t-t_0)^2}{2\tau^2} \right] \]

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Wigner delay occurs between absorption and emission processes of a single quantum dot.

Wigner delay strongly dependent on the T1-time of the quantum dot, here $T1 = (700 \pm 100)$ ps.

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Non-Markovian signatures in Wigner delays

Wigner delay induced by a single quantum dot:

Markovian theory via Lindblad-type dephasing

\[
\dot{\rho} = -\frac{i}{\hbar}[H(t), \rho] + \frac{\Gamma}{2}D[\sigma_{12}]\rho + \frac{\gamma_p}{2}D[\sigma_{22}]\rho
\]

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\dot{\rho}_{22} = -\Gamma \rho_{22} + 2\text{Im}[\Omega(t)\rho_{12}]
\]

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\dot{\rho}_{12} = (i\Delta - \Gamma/2 - \gamma_p)\rho_{12} - i\Omega(t)(2\rho_{22} - 1)
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\end{align*}
\]

Bloch equations solved in the adiabatically limit

\[
\tau_W = \frac{d\phi}{d\omega} = \frac{1}{\gamma + \Delta^2/\gamma}
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\[
\gamma = \frac{\Gamma}{2} + \gamma_p
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Choose the pure dephasing to reproduce for a fixed radiative lifetime constant

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Markovian theory fails to reproduce both limits and not the asymmetries between red- and blue-detuned Wigner delays
Non-Markovian signatures in Wigner delays

Wigner delay in the presence of electron-phonon interaction:

\[ H_{\text{dec}} = \sigma_{22} \sum_q g_{12}^q \left[ b_q^\dagger(t) + b_q(t) \right] \]

Non-Markovian theory via semiconductor Bloch equations

\[
\begin{align*}
\partial_t \langle \sigma_{22} \rangle &= -2\Gamma \langle \sigma_{22} \rangle + 2\text{Im} \left[ \Omega(t) \langle \sigma_{12} \rangle \right], \\
\partial_t \langle \sigma_{12} \rangle &= -(\Gamma + i\Delta) \langle \sigma_{12} \rangle - i\Omega(t) \left(2\langle \sigma_{22} \rangle - 1\right) \\
&\quad - i \sum_q g_{12}^q \langle b_q \sigma_{12} \rangle + g_{12}^{q*} \langle b_q^\dagger \sigma_{12} \rangle
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\]

Bloch equations solved numerically in the second-order Born level

\[
\begin{align*}
\partial_t \langle b_q \sigma_{12} \rangle &= -(\Gamma + i\Delta + i\omega_q)\langle b_q \sigma_{22} \rangle - i\Omega(t) \left(2\langle b_q \sigma_{22} \rangle - \langle b_q \rangle \right) - ig_{12}^{q*} \langle b_q^\dagger b_q \rangle \langle \sigma_{12} \rangle \\
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\]

Coupling element input parameter from material theory of InAs/GaAs (bulk phonons)

Non-Markovian theory reproduces well both limits and the asymmetries

Strauß, AC et al, PRL 122, 107401 (2019); arXiv: 1805.06357v1
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*with Nikolett Nemet, Scott Parkins*
Feedback in the quantum regime

Experiments on the single quanta level feedback coupling:
- Experiments with cold atoms
  - Dissipative dynamics of a laser-driven emitter, position dependent
  - Note kink in signal

![Graph showing time evolution of G_m(2)(T) with markers for antinode, node, and slope at different times.](image)
Feedback in the quantum regime

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  • Transmission controlled by the atom’s position at length L

Single atom-mirror:

a) Lock-in detection
   PMT 2

Probe
   \[\lambda/4\]
   Dielectric mirror

   Single atom+ mirror cavity

   \[\lambda/4\]

   Atom mirror

   \[\lambda/4\]
   PMT 3

   Lock-in detection

   PMT 1

Feedback in the quantum regime

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- Experiments with cold atoms
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  - Note kink in signal
  - Transmission controlled by the atom’s position at length \( L \)
  - Sinusoidal dependence

Single atom-mirror:

- Lock-in detection
- Single atom + mirror cavity
- PMT 2
- PMT 3
- \( g_0 \sin (kL) \)
- Photocurrent (counts/s)
- Mirror position (arb. units)

Goal: Stabilize an initial given coherence even in the presence of a reservoir at finite temperature

\[ \hat{H} = \hat{H}_S + \hat{H}_R + \hat{H}_{LB}(\hat{b}, \hat{b}^\dagger, \hat{P}_i, \hat{P}_i^\dagger) \]

\[ \hat{H}_R/\hbar = \omega_0 \hat{b}^\dagger \hat{b} + \int \left[ \omega_k \hat{r}_k^\dagger \hat{r}_k + g_k (\hat{r}_k^\dagger \hat{b} + \hat{b}^\dagger \hat{r}_k) \right] dk \]

Whalen et al, Quant. Sci. and Tech. 44008 (2017)

Nemet, AC et al, arXiv: 1902.08328
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Nemet, AC et al, arXiv: 1805.2317

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\[ \hat{H}_{LB}(t) = \hbar D[\hat{b}(t) + \hat{b}^\dagger(t)]\hat{P}^\dagger(t)\hat{P}(t) \]

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We assume a reservoir at T>0 with non-Ohmic spectral density with delay

\[ J(\omega_k) = \sin^2 \left( \frac{\omega_k T}{2} \right) e^{-i\omega_k (t-t')} \]
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Due to the linear coupling between the acoustic cavity mode and the reservoir, an exact solution exist

\[ \hat{b}(t) = F(t) \hat{b}(0) + \int G_k(t) \hat{r}_k(0) dk \]

In the linear regime, the system dynamics can be exactly evaluated via a Feynman-Vernon influence functional or Suzuki-Trotter expansion

Nemet, AC et al, arXiv: 1805.2317

Nemet, AC et al, arXiv: 1902.08328
Bypassing decoherence via quantum feedback

With given initial conditions, the dynamics can be evaluated

\[ \hat{\rho}_P(t) = \exp \left\{ \left( -i \int_0^t \hat{B}(t_1) dt_1 - \frac{1}{2} \int_0^t \int_0^{t_1} [\hat{B}(t_1), \hat{B}(t_2)] dt_2 dt_1 \right) \hat{P}^\dagger(0) \hat{P}(0) \right\} \hat{\rho}_P(0) \]

Our figure of merit is the survival time of an initial introduced coherence, e.g. via an delta pulse

\[ \eta(t) = \frac{|\langle \hat{P}(t) \rangle|^2}{|\langle \hat{P}(0) \rangle|^2} \]
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Feedback stops via quantum interference the decoherence process – a synchronisation between the oscillators take place
Bypassing decoherence via quantum feedback

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\dot{\rho}_P(t) = \exp \left\{ \left( -i \int_0^t \hat{B}(t_1) dt_1 - \frac{1}{2} \int_0^t \int_0^{t_1} [\hat{B}(t_1), \hat{B}(t_2)] dt_2 dt_1 \right) \hat{P}^\dagger(0) \hat{P}(0) \right\} \dot{\rho}_P(0)
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Delay time and phase-matching allow very long coherence times initial coherence at room temperature up to 200 ps

Nemet, AC et al, arXiv: 1805.2317
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*with Leon Droenner, Nicolas Naumann, and Andreas Knorr*
Quantum feedback in the nonlinear or many photon regime

For open quantum system case dynamics, the model is too detailed in the bath description:

\[
\frac{H}{\hbar} = \omega_0 c^\dagger c + \int dk \ \omega_k \ d_k^\dagger d_k + \int dk \ g_k \sin(kL)(d_k^\dagger c + c^\dagger d_k)
\]

within the interaction picture

\[
H_I(t) = -i\hbar g_0 \left( c^\dagger \left[ \int dk (1 - e^{i2kL}) \ d_k e^{-i(\omega_k - \omega_0)t} \right] \right) - \text{h.c.}
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Pichler, Zoller PRL 116, 93601 (2016)
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\]

Integrate Schrödinger equation

\[
|\psi(t)\rangle_I = \mathcal{T} \left\{ \exp \left[ -\frac{i}{\hbar} \int_0^t \hat{H}_I(t')dt' \right] \right\} |\psi(0)\rangle_I
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and solve stroboscopically

\[
|\psi(\Delta t)\rangle_{I} = \exp \left[ -\frac{g_0}{2} c \left( \Delta R(\Delta t) + e^{i\omega_0 \tau} \Delta R(\Delta t - \tau) \right) + \text{h.c.} \right] |\psi(0)\rangle_{I}
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\]

\[
|\psi(2\Delta t)\rangle_I = \exp \left[ -\frac{g_0}{2} c \left( \Delta R(\Delta t) + e^{i\omega_0 \tau} \Delta R(\Delta t - \tau) \right) + \text{h.c.} \right] \exp \left[ -\frac{g_0}{2} c \left( \Delta R(\Delta t) + e^{i\omega_0 \tau} \Delta R(\Delta t - \tau) \right) + \text{h.c.} \right] |\psi(0)\rangle_I
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Pichler, Zoller PRL 116, 93601 (2016)
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\]

after SVD, yielding an MPS form

\[
|\Psi\rangle = \sum_{i_1 \ldots i_N} A_{i_1}^{[1]} \ldots A_{i_N}^{[N]} |i_1\rangle \ldots |i_N\rangle = \sum_i A_i |i\rangle
\]

Quantum feedback in the nonlinear or many photon regime

Schrödinger equation yields reversible dynamics.
Example: Driven and decaying two-level system.

\[ |\psi(n+1)\rangle = \exp \left[ -i \Delta t \Omega_L (\sigma^+ + \sigma^-) - \sqrt{\Gamma \Delta t} \sigma_\Delta R^\dagger(n) \right] |\psi(n)\rangle \]
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\]

Time-reversal yields initial state. Full information of the reservoir in state. Numerical exact solution and dissipatively driven-correlation included.

\[
\langle \psi(n-1) | = \langle \psi(n) | \exp \left[ i\Delta t \Omega_L \left( \sigma^+ + \sigma^- \right) - \sqrt{\Gamma \Delta t} \sigma_- \Delta R(n) \right]
\]

Pichler, Zoller PRL 116, 93601 (2016)
Lu, AC et al, PRA 63, 63840 (2017)
Selective photon-probability control

Pulsed and decaying two-level system.

Nearly perfect single photon emission for $\pi$-pulse
Selective photon-probability control

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Two-photon emission events are favored for $2\pi$-pulses.

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Droenner, AC et al, PRA 99, 23840 (2019); arXiv:1801.03342v2
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*with Leon Droenner, and Markus Heyl*
Discrete Time Crystal

Illustration of a discrete time-crystal

\[ H_F = \Omega \sum_{i=1}^{N} \sigma_i^x \]

\[ \langle M \rangle = \frac{1}{N} \sum_{i=1}^{N} (-1)^i \frac{\langle \sigma_i^z \rangle}{2} \]

If driving is perfect \( \varepsilon=0 \), the magnetization shows a single peak in the Fourier spectra. Perfect periodicity.
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Illustration of a discrete time-crystal

\[ \mathcal{H}_F = \Omega \sum_{i=1}^{N} \sigma_i^x \]

If driving is perfect \( \epsilon=0 \), the magnetization shows a single peak in the Fourier spectra. Perfect periodicity.
Discrete Time Crystal

Illustration of a discrete time-crystal

\[ \mathcal{H}_F = (\Omega - \epsilon) \sum_{i=1}^{N} \sigma_i^x \]

The spin chain of N spins returns despite imperfect rotation back to its initial state.

Figure of merit and observable (staggered magnetization):

\[ \langle M \rangle = \frac{1}{N} \sum_{i=1}^{N} (-1)^i \frac{\langle \sigma_i^z \rangle}{2} \]

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If driving is imperfect \( \epsilon > 0 \), the magnetization dynamics shows an envelope. Imperfect periodicity.

Discrete Time Crystal

Illustration of a discrete time-crystal

\[ \mathcal{H}_F = (\Omega - \epsilon) \sum_{i=1}^{N} \sigma_i^x \]
\[ \mathcal{H}_I = \sum_{i=1}^{N-1} J \sigma_i^z \sigma_{i+1}^z \]

The spin chain of \(N\) spins returns despite imperfect rotation back to its initial state.

Figure of merit and observable (staggered magnetization):


\[ \langle M \rangle = \frac{1}{N} \sum_{i=1}^{N} (-1)^i \langle \sigma_i^z \rangle \]

If driving is imperfect \(\epsilon > 0\), and interaction switched on, single peak appears but is damped due to thermalization within chain. Vanishing periodicity for large \(N\).
Discrete Time Crystal

Illustration of a discrete time-crystal

\[ \mathcal{H}_F = (\Omega - \epsilon) \sum_{i=1}^{N} \sigma_i^x \]

\[ \mathcal{H}_I = \sum_{i=1}^{N-1} J \sigma_i^z \sigma_{i+1}^z + \sum_{i=1}^{N} h_i \sigma_i^z \]

The spin chain of N spins returns despite imperfect rotation back to its initial state.

Figure of merit and observable (staggered magnetization):


\[ \langle M \rangle = \frac{1}{N} \sum_{i=1}^{N} (-1)^i \frac{\langle \sigma_i^z \rangle}{2} \]

If driving is imperfect \( \epsilon > 0 \), and interaction switched and disorder is present, thermalization is prevented. Periodicity even for large N.
Discrete Time Crystal stabilized against dissipation

Time-crystal in the presence of losses
Lazarides and Moessner, Phys. Rev. B 95, 195135 (2017)

Periodicity is lost when Markovian reservoir (bath) is coupled to the chain. Thermalization within chain is prevented due to many-body localization but thermalization with bath is inevitable.
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N=40 spins for different dissipative strengths and imperfect driving
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Even stable against imperfect quantum feedback phase
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Thank you for the attention!