Non-Markovian Quantum Control of solid-state based Qubits

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I: Semiconductor Quantum Dot

- Semiconductor physics and heterostructures
- Quantum Dot Hamiltonian and equation of motion approach
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   - Quantum Dot Hamiltonian and equation of motion approach

II: Laser-driven Quantum Dot
   - Time-resolved phonon-assisted Mollow triplet
   - Proposal for a phonon laser
OUTLINE

I: Semiconductor Quantum Dot
- Semiconductor physics and heterostructures
- Quantum Dot Hamiltonian and equation of motion approach

II: Laser-driven Quantum Dot
- Time-resolved phonon-assisted Mollow triplet
- Proposal for a phonon laser

III: Quantum Dot – cavity QED
- Enhancement of collapse and revival phenomenon
- Photon-loss induced quantum feedback
(i) semiconductor quantum dot

- Semiconductor physics and heterostructures
- Quantum Dot Hamiltonian and equation of motion approach
Semiconductor structures – effective mass approximation

\[ \mathcal{H}_0 = -\frac{\hbar^2}{2m_0} \Delta + V_{\text{lat}}(\mathbf{r}) \]

Bandstructure of GaAs:
Parabolic structure of lowest conduction and highest valence band
Semiconductor structures – effective mass approximation

\[ \mathcal{H}_0 = -\frac{\hbar^2}{2m_0} \Delta + V_{\text{lat}}(\mathbf{r}) \]

Bandstructure of GaAs:
Parabolic structure of lowest conduction and highest valence band

\[ E_n(\mathbf{k}) \approx E_n(0) + \frac{\hbar^2 k^2}{2m^*_n} \quad \text{with} \quad \frac{1}{m^*_n} = \frac{1}{\hbar^2} \frac{\partial^2 E_n(\mathbf{k})}{\partial k^2} \bigg|_{k = 0} \]

Mixing semiconductors with different band gaps: nanostructures
Confinement potential: Geometry, Size, Material specifics

\[
\left[ -\frac{\hbar^2}{2m^*_n} \Delta + V_{\text{conf}}(r) \right] \xi_n(r) = \varepsilon_n \xi_n(r) \quad \text{with} \quad \varepsilon_n = E - E_n(0)
\]
Semiconductor Nanostructures and second quantization

Confinement potential: Geometry, Size, Material specifics

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\[ \varphi_n(\mathbf{r}) = u_{n,k \approx 0}(\mathbf{r}) \xi_n(\mathbf{r}) \]

\[ \hat{\Psi}(\mathbf{r}) = \sum_n \varphi_n(\mathbf{r}) \hat{a}_n, \quad \hat{\Psi}^\dagger(\mathbf{r}) = \sum_n \varphi_n^*(\mathbf{r}) \hat{a}_n^\dagger \]
**Confinement potential:**
Geometry, Size, Material specifics

\[
\left[ -\frac{\hbar^2}{2m_n^*} \Delta + V_{\text{conf}}(\mathbf{r}) \right] \xi_n(\mathbf{r}) = \varepsilon_n \xi_n(\mathbf{r}) \quad \text{with} \quad \varepsilon_n = E - E_n(0)
\]

**Microscopic calculated Wave function for 2nd quantization**

\[
\phi_n(\mathbf{r}) = u_{n,k\approx0}(\mathbf{r}) \xi_n(\mathbf{r}) \quad \hat{\Psi}(\mathbf{r}) = \sum_n \phi_n(\mathbf{r}) \hat{a}_n, \quad \hat{\Psi}^\dagger(\mathbf{r}) = \sum_n \phi_n^*(\mathbf{r}) \hat{a}_n^\dagger
\]

\[
\hat{H}_0^c = \int d^3 r \ \hat{\Psi}^\dagger(\mathbf{r}) \left( -\frac{\hbar^2}{2m_0} \Delta + V_{\text{lat}}(\mathbf{r}) + V_{\text{conf}}(\mathbf{r}) \right) \hat{\Psi}(\mathbf{r})
\]

\[
= \sum_n \varepsilon_n \hat{a}_n^\dagger \hat{a}_n.
\]
Semiconductor quantum dots

Quantum dots: An artificial atom

Discrete energy levels
→ Optical properties by design
→ Electrical pumping possible

Shields, Nat. Photonics, 221 (2007)
**Semiconductor quantum dots**

Quantum dots: An artificial atom

- Discrete energy levels
- Optical properties by design
- Electrical pumping possible

But:
- Semiconductor environment (wetting layer, phonons) leads to dephasing!
Dephasing mechanism in a semiconductor QD

Pure dephasing:

→ Deformation (LA) potential

\[ g_{LA,q}^{\lambda,\mu,3D} = \delta_{\lambda,\mu} \sqrt{\frac{\hbar q}{2 \rho c_s V}} D_{\lambda} \]
Dephasing mechanism in a semiconductor QD

Pure dephasing:

→ Deformation (LA) potential\(^1\)

\[
\gamma_{\lambda,\mu,3D} = \delta_{\lambda,\mu} \sqrt{\frac{\hbar q}{2\rho c_s V}} D_\lambda
\]

1\(^\text{PRB 83, 041304(R) (2011)}\)
Pure dephasing:

→ Fröhlich (LO) potential\(^1\)

\[ g_{\text{LO},q}^{\mu,3D} = \frac{1}{q} \sqrt{\frac{e_0^2 \hbar \omega_{\text{LO}}}{2 \varepsilon_0 V}} \left( \frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon_\text{st}} \right) \]

\(^1\text{PRB 83, 041304(R) (2011)}\)
Dephasing mechanism in a semiconductor QD

Pure dephasing:

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\[ S_{LO,q}^{\lambda \mu,3D} = \frac{1}{q} \sqrt{\frac{e^2 \hbar \omega_{LO}}{2\varepsilon_0 V}} \left( \frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon_{st}} \right) \]

\[ 1^\text{PRB} 83, 041304(R) (2011) \]
GOAL

Developing a theoretical framework to find advantageous features of the semiconductor environment
(i) semiconductor quantum dot

- Semiconductor physics and heterostructures
- Quantum Dot Hamiltonian and equation of motion approach
Semiconductor QD cavity-QED Hamiltonian

\[ H = H_{el} + H_{phonon} + H_{photon} + H_{laser} \]

\[ H_{el} = \hbar \sum_i \omega_i a_i^\dagger a_i + \hbar \sum_{ijlm} V_{lm}^{ij} a_i^\dagger a_j^\dagger a_l a_m \]

\[ H_{phonon} = \hbar \sum_q \omega_q b_q^\dagger b_q + \hbar \sum_{ij,q} g_{ij}^{q} a_i^\dagger a_j b_q^\dagger + H.c. \]

\[ H_{photon} = \hbar \sum_k \omega_k c_k^\dagger c_k + \hbar \sum_{ij,k} M_{ij}^{k} a_i^\dagger a_j c_k^\dagger + H.c. \]

\[ H_{laser} = \hbar \sum_{ij} \Omega_{ij} a_i^\dagger a_j + H.c. \]
Semiconductor QD cavity-QED Hamiltonian

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\[ H_{el} = \hbar \sum_i \omega_i a_i^\dagger a_i + \hbar \sum_{ijlm} V_{lm}^{ij} a_i^\dagger a_j^\dagger a_l a_m \]

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\[ H_{photon} = \hbar \sum_k \omega_k c_k^\dagger c_k + \hbar \sum_{i,j,q} M_{q}^{ij} a_i^\dagger a_j c_k^\dagger + H.c. \]

\[ H_{laser} = \hbar \sum_{i,j} \Omega_{ij} a_i^\dagger a_j + H.c. \]
for every possible combination of phonon, photon, and electron operators for example a two-level system:

\[
G_{m,n}^{p,s} := a_v^\dagger a_v c^\dagger p c^s b^\dagger m b^n
\]

\[
E_{m,n}^{p,s} := a_c^\dagger a_c c^\dagger p c^s b^\dagger m b^n
\]

\[
T_{m,n}^{p,s} := a_v^\dagger a_c c^\dagger p c^s b^\dagger m b^n
\]
Using product rule for operators:

\[
\partial_t \left( a_c^\dagger a_c c^\dagger c b_q^\dagger b_q \right) = \left( \partial_t a_c^\dagger a_c c^\dagger c \right) b_q^\dagger b_q + c^\dagger c \left( \partial_t a_c^\dagger a_c b_q^\dagger b_q \right)
\]

and generalized commutation relations:

\[
[A, F(B)] = [A, B]F'(B)
\]

for every possible combination of phonon, photon, and electron operators for example a two-level system:
Using product rule for operators:  
\[ \partial_t \left( a_c^{\dagger} a_c c^\dagger c b_q^{\dagger} b_q \right) = \left( \partial_t a_c^{\dagger} a_c c^\dagger c \right) b_q^{\dagger} b_q + c^\dagger c \left( \partial_t a_c^{\dagger} a_c b_q^{\dagger} b_q \right) \]

and generalized commutation relations:

\[ [A, F(B)] = [A, B]F'(B) \]

\[ G_{m,n}^{p,s} := a_v^{\dagger} a_v c^\dagger p c^s b^{\dagger} m b^n \]

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for every possible combination of phonon, photon, and electron operators for example a two-level system:

and

their dynamics, e.g.

\[ \partial_t \langle T_{m,n}^{p,s} \rangle = \]

\[ = -i \left[ \omega_{cv} - (p - s)\omega_0 - (m - n)\omega_{LO} - i(p + s)\kappa - i\gamma \right] \langle T_{m,n}^{p,s} \rangle \]

\[ - i p \left( \langle E_{m,n}^{p-1,s} \rangle - \langle E_{m,n}^{p,s+1} \rangle \right) - i M \left( \langle G_{m,n}^{p,s+1} \rangle - \langle G_{m,n}^{p,s} \rangle \right) - i \Omega(t) \left( \langle E_{m,n}^{p,s} \rangle - \langle G_{m,n}^{p,s} \rangle \right) \]

\[ - i \left( \langle T_{m,n+1}^{p,s} \rangle - i \langle T_{m,n+1}^{p,s} \rangle \right) + i m g_v \langle T_{m-1,n}^{p,s} \rangle - i n g_c \langle T_{m,n-1}^{p,s} \rangle, \]
For example, in the case of LO-phonon assisted vacuum Rabi oscillations \( E_{00}^{11} = 0 \):

\[
\begin{align*}
E_{00}^{00} & \rightarrow E_{00}^{11} & \text{Photon interaction:} \\
T_{00}^{10} & \rightarrow T_{00}^{00} & \text{numerically solvable up to arbitrary accuracy,}
\end{align*}
\]

reproducing analytical solutions of the IBM and JCM.
For example, in the case of LO-phonon assisted vacuum Rabi oscillations ($E_{00}^{11} = 0$):

- **Phonon interaction:**
  - $E_{00}^{20} \rightarrow T_{10}^{10} \rightarrow E_{00}^{10} \rightarrow T_{10}^{10} \rightarrow E_{00}^{00}$
  - $E_{00}^{01}$ \rightarrow $E_{00}^{02}$

- **Photon interaction:**
  - $E_{00}^{00} \rightarrow P_{00}^{11} \rightarrow T_{11}^{10} \rightarrow P_{11}^{10} \rightarrow E_{00}^{00}$
  - $E_{00}^{11}$ \rightarrow $E_{00}^{11}$

Numerically solvable up to arbitrary accuracy, reproducing analytical solutions of the IBM and JCM.
(ii) Laser-driven quantum dot

- Time-resolved phonon-assisted Mollow triplet
- Proposal for a phonon laser
Strong excitation:

\[ |\tilde{c}, n_q\rangle \]

\[ |\tilde{v}, n_q\rangle \]

\[ \omega_l \]

\[ \omega_k \]

(b) strong excitation
Strong excitation:

1. Laser pulse excites the sample.
2. The field $E(t)$ is filtered by $F_s(t)$.
3. The detector reads the output.

(b) Strong excitation:

- Initial state $|\tilde{c}, n_q\rangle$.
- Excitation by laser at $\omega_l$.
- Final state $|\tilde{\psi}, n_q\rangle$.

Energy and time spectroscopy:

1PRB 84, 075314 (2011)
Phonon coupling strength via anti-crossing

- Spectrum shows the usual Mollow triplet and phonon-assisted Mollow triplets.
- Additional anticrossings, when the Rabi-energy matches the phonon energy (Here 36.4 meV for InGaAs/GaAs-QD).
- These anti-crossings scale with the electron-phonon coupling strength.
(ii) Laser-driven quantum dot

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Proposal for a phonon laser

Non-resonant excited 2-level system in an acoustic cavity
Proposal for a phonon laser

Non-resonant excited 2-level system in an acoustic cavity

High orders of phonon operators become important $^{1,2,3}$

$^{1}$PRL 104, 156801 (2010), $^{2}$PSS(b) 248, 872 (2011), $^{3}$submitted (2012)
Generation of coherent phonons

\[ g_{ph}^{(2)}(0) \]

\[ n \]

[Graph showing the generation of coherent phonons as a function of \( \Omega [\text{meV}] \).]
Generation of coherent phonons

\( g_{ph}^{(2)}(0) \), phonon number

\( n \)

\( \Omega [\text{meV}] \)

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(iii) quantum dot cavity-QED

- Enhancement of collapse and revival phenomenon
- Photon-loss and induced quantum feedback
LA-phonon assisted collapse and revival

Coherent state: cavity field prepared initially in Glauber states
LA-phonon assisted collapse and revival

Coherent state: cavity field prepared initially in Glauber states

Markovian theory
LA-phonon assisted collapse and revival

Coherent state: cavity field prepared initially in Glauber states

Non-Markovian theory

Collapse and revival phenomenon is enhanced due to LA-phonon induced dephasing\(^1\)

\(^1\) submitted (2012)
(iii) quantum dot cavity-QED

- Enhancement of collapse and revival phenomenon
- Photon-loss and induced quantum feedback
Photon-loss induced quantum feedback

Set-up for quantum feedback

External mirror shapes the mode continuum in front of the mirror to introduce a delay effect\textsuperscript{1}

\[ H = \sum_{q} (G_q c^\dagger d_q + G^*_q d^\dagger_q c) \]

\textsuperscript{1} in preparation (2012)
Photon-loss induced quantum feedback

\[ G_q = 0 \]

Jaynes-Cummings model solution, if no outcoupling is present
Photon-loss induced quantum feedback

$G_q = G_0 \neq 0$

$G_0 \gg M$

Weak coupling solution, if outcoupling is present and stronger than light coupling
Photon-loss induced quantum feedback

\[ G_q = G_0 \sin (qL) \]
Photon-loss induced quantum feedback

External mirror shapes the mode continuum in front of the mirror to introduce a delay effect\(^1\)

\(^1\) in preparation (2012)
Alexander Carmele: Non-Markovian Quantum Control of solid-state Qubits

Graduate Lecture SS 2012

Photon-loss induced quantum feedback

Classical limit\(^1\):

\[ \langle a^\dagger_c a_c c^\dagger c \rangle \approx \langle a^\dagger_c a_c \rangle \langle c^\dagger c \rangle \]

\(^1\) in preparation (2012)
(iv) conclusion
Conclusion

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Thank you for your attention !!