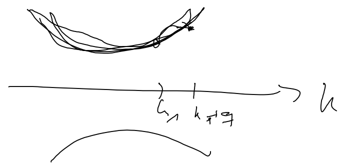
Polaron-Mastergleichung (Julian)

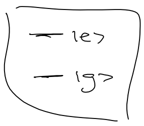
Donnerstag, 22. November 2018 16:13

$$H_{el-ph} = \sum_q \sum_k a^\dagger_{k+q} a_k \left(g_q b_q + g^*_{-q} b^\dagger_q \right)$$



$$\{a_l, a^\dagger_k\}_+ = \delta_{lk} \quad , \quad [b_p, b^\dagger_q]_- = \delta_{pq}$$

$$H^{IBM}_{int} = a^\dagger a \sum_q g_q (b_q + b^\dagger_q) \quad , \quad H_0 = H_S + H_B = \omega_e a^\dagger a + \sum_q \omega_q b^\dagger_q b_q$$



$$g^e_q - g^{hr}_q$$

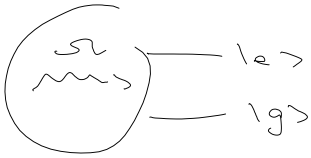
$$a^\dagger = |e\rangle\langle g|$$

$$a = |g\rangle\langle e|$$

$$\Rightarrow a^\dagger a = |e\rangle\langle e|$$

$$\sigma_{ij} = |i\rangle\langle j|$$

$$\boxed{H_{int} = \sigma_{ee} \sum_q g_q (b_q + b^\dagger_q)}$$



$$H_S = \Delta \sigma_{ee} + \Omega_L (\sigma_{eg} + \sigma_{ge})$$

$$H_B = \sum_q \omega_q b^\dagger_q b_q$$

Polaron transformation

$$H' = e^P H e^{-P} \quad , \quad P = \sigma_{ee} \sum_q \frac{g_q}{\omega_q} (b^\dagger_q - b_q)$$

$$e^X Y e^{X} = \sum_{n=0}^{\infty} \frac{1}{n!} [X, Y]_n$$

$$[X, Y]_n = [X, [X, Y]_{n-1}] \quad , \quad [X, Y]_0 = Y$$

$$[P, H_B]_1 = \left[\sigma_{ee} \sum_{q'} \frac{g_{q'}}{\omega_{q'}} (b^\dagger_{q'} - b_{q'}) , \sum_q \omega_q b^\dagger_q b_q \right]$$

$$= \sigma_{ee} \sum_{q'q} \frac{g_{q'}}{\omega_{q'}} \omega_q \left(\underbrace{[b^\dagger_{q'}, b^\dagger_q b_q]} - \underbrace{[b_{q'}, b^\dagger_q b_q]} \right)$$

$$= \sigma_{ee} \sum_q g_q (b_q - b_q^\dagger) = -H_{int} = \underline{\underline{-[P, H_{int}]_0}}$$

$$\underline{\underline{[P, H_B]_2}} = -\sigma_{ee} \sum_{q,q'} \frac{g_{q'}}{\omega_{q'}} g_q \underbrace{[b_{q'}^\dagger - b_{q'}, b_q + b_q^\dagger]}_{-2\delta_{q,q'}} = \sigma_{ee} \underbrace{\sum_q 2 \frac{g_q^2}{\omega_q}}_{2\Delta_p}$$

$$[P, H_B]_3 = 0 = [P, H_{int}]_2$$

$$[P, H_{int}]_1 = -[P, H_B]_2 = -2\Delta_p \sigma_{ee}$$

$$[P, H_S]_1 = \sum_q \frac{g_q}{\omega_q} [\sigma_{ee}, \Omega(\sigma_{eg} + \sigma_{ge})] (b_q^\dagger - b_q)$$

$$= \Omega (\sigma_{eg} - \sigma_{ge}) \sum_q \frac{g_q}{\omega_q} (b_q^\dagger - b_q)$$

$$[P, H_S]_n = \Omega (\sigma_{eg} + (-1)^n \sigma_{ge}) \left\{ \sum_q \frac{g_q}{\omega_q} (b_q^\dagger - b_q) \right\}^n$$

$$B_\pm = \exp\left\{ \pm \sum_q \frac{g_q}{\omega_q} (b_q - b_q^\dagger) \right\}$$

Collect everything:

$$H' = \Omega (\sigma_{eg} \hat{B}_- + \sigma_{ge} \hat{B}_+) + (\Delta - \Delta_p) \sigma_{ee} + \sum_q \omega_q b_q^\dagger b_q$$

$$H' = (\Delta - \Delta_p) \sigma_{ee} + \Omega \langle B \rangle (\sigma_{eg} + \sigma_{ge}) + \sum_q \omega_q b_q^\dagger b_q$$

$$H_{int}^e \rightarrow + \Omega (\sigma_{eg} + \sigma_{ge}) (B_+ + B_- - 2\langle B \rangle)$$

$$H_{int}^o \rightarrow + \Omega (\sigma_{eg} - \sigma_{ge}) (B_+ - B_-)$$

$$\langle B \rangle = \langle B_+ \rangle = \langle B_- \rangle \text{ thermal expectation value}$$

$$\dot{\tilde{\rho}}_S = -i \mathrm{Tr}_B [\tilde{H}_I', \rho(0)] - \int^t dt' \mathrm{Tr}_B [\tilde{H}_I(t), [\tilde{H}_I(t'), \rho(t')]]$$

$$\dot{\tilde{\rho}}_S = \underbrace{-i\,\mathrm{Tr}_B\left[\tilde{H}_I', \rho(0)\right]}_{\sim\, \mathrm{Tr}_B\left(\rho_B (B_+ + B_- - 2\langle B\rangle)\right)} - \int_0^t dt'\,\mathrm{Tr}_B\left[\tilde{H}_I(t), \left[\tilde{H}_I(t'), \rho(t')\right]\right]$$

$$\langle B_+\rangle = \langle B_-\rangle = \langle B\rangle$$

$$\dot{\rho}_S(t) = -i\left[H_S', \rho_S(t)\right] - \int_0^t d\tau \sum_{m=e,o} \Big(G_m(\tau)\left[S_m, e^{-iH_S'\tau} S_m e^{iH_S'\tau}\rho_S(t-\tau)\right] + H.c.\Big)$$

$$G_m(\tau) \leftarrow \mathrm{Tr}_B\left(\rho_B B_e B_e(\tau)\right)$$

$$H_{int}' = \underbrace{(\sigma_{eg}+\sigma_{ge})}_{S_e}\underbrace{(B_+ + B_- - 2\langle B\rangle)}_{B_e} + \underbrace{(\sigma_{eg}-\sigma_{ge})}_{S_o}\underbrace{(B_+ - B_-)}_{B_o}$$

Check Positivity!

$$\tilde{H}' = e^{iH_0 t} H' e^{-iH_0 t}$$

If $\Omega = \Omega(t) \Rightarrow H_S = H_S(t)$

$$H_0 = \Delta\sigma_{ee} + \Omega(t)(\sigma_{eg}+\sigma_{ge})$$

$$U = T_{\rightarrow} \exp\left[-i\int_0^t dt'\, H_0(t')\right]$$

Reference: McCutcheon and Nazir, New Journal of Physics \bf{12} 113042 (2010)