Available online at www.sciencedirect.com
ScienceDirect
Physica E 40 (2008) 1157–1159
PHYSICA E
www.elsevier.com/locate/physe

Phonon emission in two levels quantum dots

Rafael Sánchez[a,*], Gloria Platero[a], Tobias Brandes[b]

[a] Instituto de Ciencia de Materiales, CSIC, Cantoblanco, Madrid 28049, Spain
[b] Institut für Theoretische Physik, Technische Universität Berlin, D-10623 Berlin, Germany

Available online 31 August 2007

Abstract

We study the counting statistics of electron tunneling and phonon emission events in a two level quantum dot under the effect of a driving field. By tuning the intensity of the driving field, the phonon fluctuations can be taken from the expected bosonic bunching to anti-bunching. We find a configuration where this system can be used as a probe to the phonon spontaneous emission rate.

PACS: 73.63.Kv; 72.70.+m; 78.67.Hc; 42.50.Lc

Keywords: Quantum dot; Electron; Phonon; Counting statistics

In the last years, the study of electron transport through semiconductor devices has given increasing importance to the information that can be obtained from the statistics of the fluctuations in the current signal, both theoretically [1,2] and experimentally [3]. For that purpose, techniques that were previously developed for the study of photon emission in illuminated atoms [4] have been adopted. In those Quantum Optics works, the anti-bunching of photons emitted from a closed two-level atom under a resonant field (resonance fluorescence [5]), were analyzed.

Here, we consider a hybrid problem were the resonance fluorescence is studied in an open system such as a two level quantum dot (QD) connected to fermionic leads [6]. The lattice vibrations induce, at low temperatures, relaxation processes from the upper to the lower electronic state by spontaneous phonon emission. The emitted phonons play the role of the fluorescent photons. Therefore, we can extract information from both phonon emission and electron tunneling events statistics [9]. In particular, we find that phonon noise can be tuned back and forth between sub- and super-Poissonian character by using the intensity of the AC driving field.

We assume the Coulomb repulsion inside the QD to be so large that only single occupation is allowed (*Coulomb blockade* regime). A time-dependent AC field with a frequency ω drives the transition between the two levels ε_1, ε_2 close to resonance, $\Delta_\omega = \varepsilon_2 - \varepsilon_1 - \omega \approx 0$, which allows us to assume the rotating wave approximation. Thus, the electron in the QD is coherently delocalized between both levels performing *photon-assisted Rabi oscillations*. We will consider a special configuration where the emitter is coupled only to the upper level, while the collector can only receive electrons coming from the lower level. For simplicity, we consider spinless electrons.

The components of the density matrix $\rho(t) = \sum_{n_e,n_{ph}} \rho^{(n_e,n_{ph})}(t)$ give the probability that, during a certain time interval t, n_e electrons have tunneled out of a given electron–phonon system and n_{ph} phonons have been emitted [7]. We define the generating function (GF) [8] $G(t, s_e, s_{ph}) = \sum_{n_e,n_{ph}} s_e^{n_e} s_{ph}^{n_{ph}} \rho^{(n_e,n_{ph})}(t)$, where $s_{e(ph)}$ are the electron (phonon) counting variables whose derivatives give us the correlations:

$$\frac{\partial^{p+q}\,\mathrm{tr}G(t,1,1)}{\partial s_e^p\,\partial s_{ph}^q} = \left\langle \prod_{i=1}^{p}\prod_{j=1}^{q}(n_e - i + 1)(n_{ph} - j + 1)\right\rangle. \quad (1)$$

Thus, we are able to obtain the mean number $\langle n_\alpha\rangle$, the variance $\sigma_\alpha^2 = \langle n_\alpha^2\rangle - \langle n_\alpha\rangle^2$ (which give the $\alpha = \mathrm{e,ph}$ current

*Corresponding author.
E-mail address: rafael.sanchez@icmm.csic.es (R. Sánchez).

1386-9477/$ - see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.physe.2007.08.025

and noise, respectively), or define the correlation between the electron and phonon counts, $\langle n_{\rm e} n_{\rm ph}\rangle$.

We integrate the equations of motion for the GF:

$$\dot{G}(t, s_{\rm e}, s_{\rm ph}) = M(s_{\rm e}, s_{\rm ph}) G(t, s_{\rm e}, s_{\rm ph}), \tag{2}$$

that generalizes the Master equation, $\dot{\rho}(t) = M(1,1)\rho(t)$, by introducing the counting variables in those terms corresponding to the tunneling of an electron to the collector lead and the emission of a phonon.

The long-time behaviour is extracted from the pole near $z = 0$ in the Laplace transform of the GF, $\tilde{G}(z, s_{\rm e}, s_{\rm ph}) = (z - M)^{-1}\rho(0)$, where $\rho(0)$ is the initial state. From the Taylor expansion of the pole $z_0 = \sum_{m,n>0} c_{mn}(s_{\rm e}-1)^m (s_{\rm ph}-1)^n$, we obtain $G(t, s_{\rm e}, s_{\rm ph}) \sim g(s_{\rm e}, s_{\rm ph}) e^{z_0 t}$ and the central moments:

$$\langle n_{\rm e(ph)}\rangle = \frac{\partial g(1,1)}{\partial s_{\rm e(ph)}} + c_{10(01)} t, \tag{3a}$$

$$\sigma^2_{\rm e(ph)} = \frac{\partial^2 g(1,1)}{\partial s^2_{\rm e(ph)}} - \left(\frac{\partial g(1,1)}{\partial s_{\rm e(ph)}}\right)^2 + (c_{10(01)} + 2c_{20(02)})t. \tag{3b}$$

Higher moments or electron–phonon correlations, which are not discussed here, are straightforwardly obtained. In the large time asymptotic limit, all the information is included in the coefficients c_{mn}. Then, the current is $I_{\rm e(ph)} = \partial\langle n_{\rm e(ph)}\rangle/\partial t = c_{10(01)}$. The Fano factor is $F_{\rm e(ph)} = 1 + 2c_{20(02)}/c_{10(01)}$ so the sign of the second term in the r.h.s. defines the sub-($F<1$) or super-($F>1$) Poissonian character of the noise. We describe our system by the Hamiltonian: $\hat{H}(t) = \sum_i \varepsilon_i \hat{d}_i^\dagger \hat{d}_i + (\Omega/2)({\rm e}^{-{\rm i}\omega t}\hat{d}_2^\dagger \hat{d}_1 + h.c.) + \sum_Q \omega_Q \hat{a}_Q^\dagger \hat{a}_Q + \sum_Q \lambda_Q (\hat{d}_2^\dagger \hat{d}_1 \hat{a}_Q + h.c.) + \sum_{k\alpha i} V_{i\alpha}(\hat{c}_{k\alpha}^\dagger \hat{d}_i + h.c.) + \sum_{k\sigma} \varepsilon_{k\alpha} \hat{c}_{k\alpha}^\dagger \hat{c}_{k\alpha}$, where $\hat{a}_Q$, $\hat{c}_{k\alpha}$, and $\hat{d}_i$ are annihilation operators of phonons and electrons in the leads and in the QD, respectively. The spontaneous emission and electron tunneling processes are described by the rates: $\gamma = 2\pi|\lambda_{\varepsilon_2-\varepsilon_1}|^2$and $\Gamma_\alpha = 2\pi|V_\alpha|^2$, respectively. In our configuration, $\Gamma_{2R} = \Gamma_{1L} = 0$ and, for simplicity, $\Gamma_{2L} = \Gamma_{1R} = \Gamma$. Writing the density matrix as a vector, $\rho = (\rho_{00}, \rho_{11}, \rho_{12}, \rho_{21}, \rho_{22})^{\rm T}$, the equation of motion of the GF (2) is described, in the Born–Markov approximation, by the matrix:

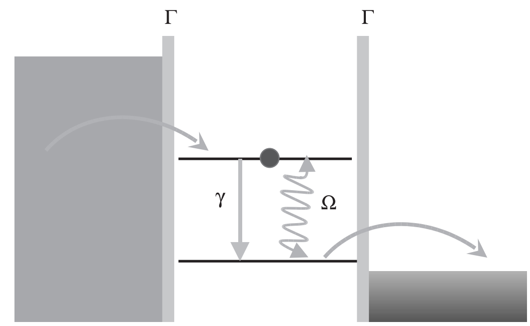
$$M = \begin{pmatrix} -\Gamma & \Gamma s_{\rm e} & 0 & 0 & 0 \\ 0 & -\Gamma & {\rm i}\frac{\Omega}{2} & -{\rm i}\frac{\Omega}{2} & \gamma s_{\rm ph} \\ 0 & {\rm i}\frac{\Omega}{2} & -\frac{\tilde{\Gamma}}{2} + {\rm i}\Delta_\omega & 0 & -{\rm i}\frac{\Omega}{2} \\ 0 & -{\rm i}\frac{\Omega}{2} & 0 & -\frac{\tilde{\Gamma}}{2} - {\rm i}\Delta_\omega & {\rm i}\frac{\Omega}{2} \\ \Gamma & 0 & -{\rm i}\frac{\Omega}{2} & {\rm i}\frac{\Omega}{2} & -\gamma \end{pmatrix}, \tag{4}$$

where Ω is the Rabi frequency, which is proportional to the intensity of the AC field, and $\tilde{\Gamma} = \Gamma + \gamma$, see also Scheme 1.

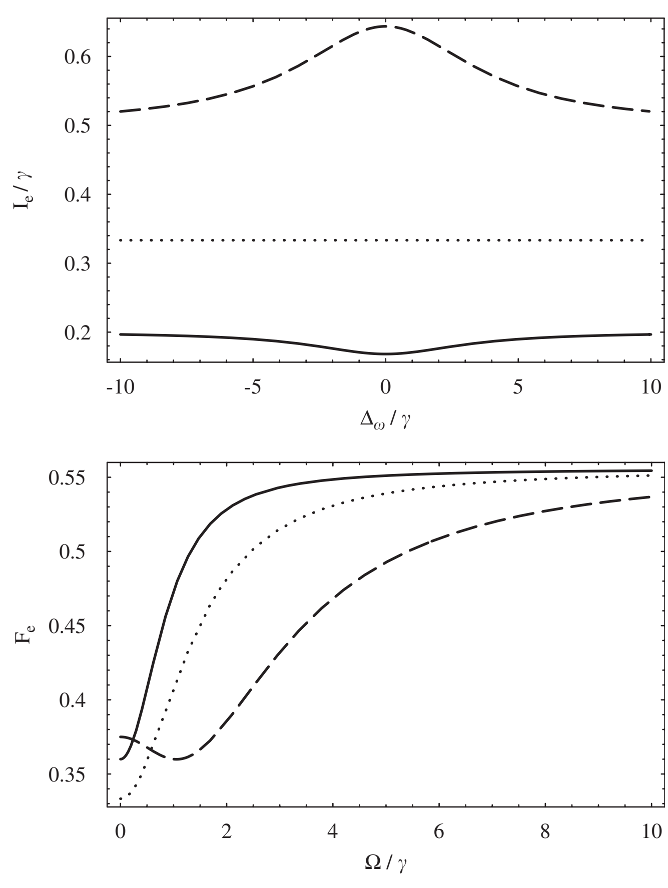
The obtained mean electron and phonon currents are found to be proportional. In resonance ($\Delta_\omega = 0$):

$$\frac{1}{\Gamma} I_{\rm e} = \frac{1}{\gamma} I_{\rm ph} = \frac{\gamma\tilde{\Gamma} + \Omega^2}{\tilde{\Gamma}(\Gamma + 2\gamma) + 3\Omega^2}. \tag{5}$$

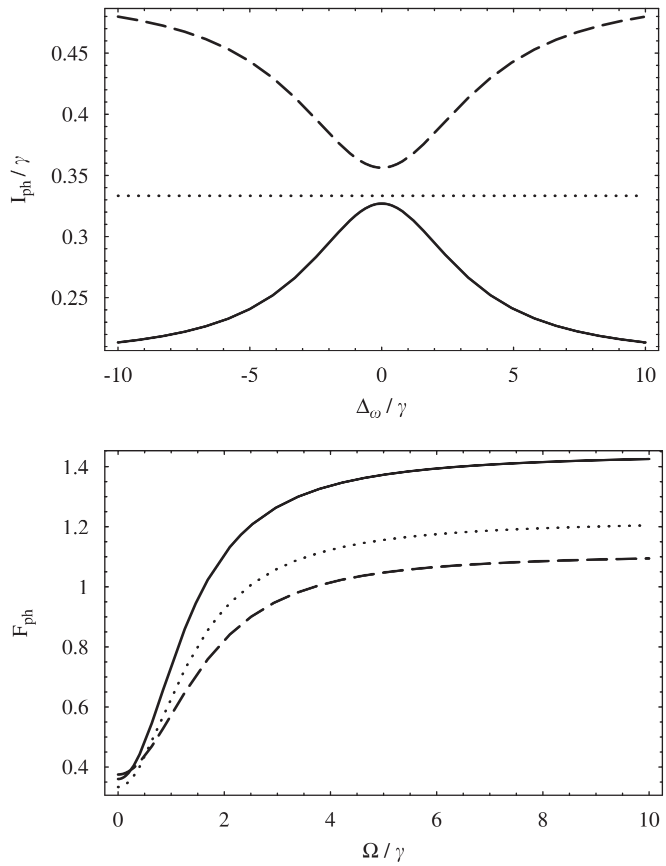
However, a more interesting result is its dependence with the detuning: $(\partial\langle n_{\rm e(ph)}\rangle/\partial\Delta_\omega) \propto \Gamma - \gamma$. As a consequence, both electronic (cf. Fig. 1) and phononic (cf. Fig. 2) currents show a resonant to antiresonant crossover which allows to extract information on the spontaneous phonon



Scheme 1. Schematic diagram of our system and the processes involved, see text.



Scheme 1. Electronic current and Fano factor as a function of the detuning (for $\Omega = 5\gamma$) and the field intensity (in resonance), respectively. The different curves correspond to different tunneling rates: $\Gamma = \gamma/2$ (solid), $\Gamma = \gamma$ (dotted), and $\Gamma = 2\gamma$ (dashed).



Scheme 2. Phononic current and Fano factor as a function of the detuning (for $\Omega = 5\gamma$) and the field intensity (in resonance), respectively. The different curves correspond to different tunneling rates: $\Gamma = \gamma/2$ (solid), $\Gamma = \gamma$ (dotted), and $\Gamma = 2\gamma$ (dashed).

emission rate (which depends on the sample) by externally modifying the tunneling couplings.

The electron Fano factor in resonance:

$$F_e = \frac{\tilde{\Gamma}^2(2\gamma^2+\Gamma^2)+2(3\gamma^2+6\Gamma\gamma-\Gamma^2)\Omega^2+5\Omega^4}{(3\Omega^2+\tilde{\Gamma}(\Gamma+2\gamma))^2} \tag{6}$$

is sub-Poissonian, as expected (see Fig. 1). On the other hand, the phononic noise shows a transition, for a given driving intensity, from *fermionic-like* sub-Poissonian values (when the field intensity is weak compared to the tunneling rate and the emission of phonons is strongly correlated with the electronic current) to the expected super-Poissonian character:

$$F_{ph} = 1 - \frac{2\gamma(\Gamma\tilde{\Gamma}^2(\gamma+2\Gamma)-(\gamma^2-6\Gamma\gamma+\Gamma^2)\Omega^2-\Omega^4)}{\Gamma(\tilde{\Gamma}(\Gamma+2\gamma)+3\Omega^2)^2}, \tag{7}$$

as can be seen in Fig. 2.

To summarize, we have studied the current and noise properties for both the transferred electrons and spontaneously emitted phonons from a two level QD. If the emitter(collector) is not connected to the lower(upper) level of the dot, a crossover from resonance to antiresonance in the electronic and phononic currents can serve as a probe for the spontaneous emission rate. We also show how the character of the phononic noise can be tuned between sub- and super-Poissonian by the manipulation of the driving field intensity.

Work supported by the M.E.C. of Spain through Grant No. MAT2005-00644 and the UE network Grant No. MRTN-CT-2003-504574.

References

[1] L.S. Levitov, G.B. Lesovik, Pis'ma Zh. Eksp. Teor. Fiz. 58 (1993) 225 [JETP Lett. 58 (1993) 230];
D.A. Bagrets, Yu.V. Nazarov, Phys. Rev. B 67 (2003) 85316.
[2] Ya.M. Blanter, M. Büttiker, Phys. Rep. 336 (2000) 1;
S. Kohler, et al., Phys. Rep. 406 (2005) 379.
[3] S. Gustavsson, et al., Phys. Rev. Lett. 96 (2006) 76605;
S. Gustavsson, et al., Phys. Rev. B 74 (2006) 195305.
[4] R.J. Cook, Phys. Rev. A 23 (1981) 1243.
[5] L. Mandel, Opt. Lett. 4 (1979) 205.
[6] R. Aguado, T. Brandes, Phys. Rev. Lett. 92 (2004) 206601.
[7] M.B. Plenio, P.L. Knight, Rev. Mod. Phys. 70 (1998) 101;
S.A. Gurvitz, Ya.S. Prager, Phys. Rev. B 53 (1996) 15932.
[8] D. Lenstra, Phys. Rev. A 26 (1982) 3369.
[9] R. Sánchez, et al., Phys. Rev. Lett. 98 (2007) 146805.