Quantum Communication and Teleportation

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- Discussing the results

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Recall: Composite Quantum Systems

- consider two 2-dimensional systems (qubits) with the Hilbert-spaces $\mathcal{H}_A$ (Alice's system) and $\mathcal{H}_B$ (Bob's system)
- space of composite system is $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$
- basis of $\mathcal{H}_{AB}$ is formed by the tensor product of the basis vectors of the subsystems A and B;
  e.g. the basis
  $\{|0\rangle_A \otimes |0\rangle_B, |0\rangle_A \otimes |1\rangle_B, |1\rangle_A \otimes |0\rangle_B, |1\rangle_A \otimes |1\rangle_B\}$ in the 2-qubit-system ; with

$$|0\rangle_A \otimes |0\rangle_B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}_A \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}_B = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}_{AB}$$

Recall: Tensor product

- multiplication with scalars is commutative
- linear in $\mathcal{H}_A$ and respectively in $\mathcal{H}_B$
- notations: $|0\rangle_A \otimes |0\rangle_B \equiv |0\rangle_A |0\rangle_B \equiv |00\rangle_{AB} \equiv |00\rangle$
- superposition in composite system: $|\psi\rangle = A_{00}|00\rangle + A_{01}|01\rangle + A_{10}|10\rangle + A_{11}|11\rangle$, with $\sum |A_{ij}|^2 = 1$

$$P \otimes Q = \begin{pmatrix} p_{11}Q & \cdots & p_{1m}Q \\ \vdots & \ddots & \vdots \\ p_{n1}Q & \cdots & p_{nm}Q \end{pmatrix}$$

for vectors ($m \times 1$, rank-1-tensor) we can use the dyadic product and obtain a rank-2-tensor:

$$\mathbf{a} \otimes \mathbf{b} = \begin{pmatrix} a_1 \\ \vdots \\ a_m \end{pmatrix} \begin{pmatrix} b_1 & \cdots & b_m \end{pmatrix} = \begin{pmatrix} a_1 b_1 & \cdots & a_1 b_m \\ \vdots & & \vdots \\ a_m b_1 & \cdots & a_m b_m \end{pmatrix}$$

- dimensions are multiplicative (not additive as in classical systems, where we use the direct sum, when particles join) for vectors ($m \times 1$, rank-1-tensor) we can use the dyadic product and obtain a rank-2-tensor:

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Separable or entangled

Regarding quantum communication we are interested in whether a system is entangled or separable.
for pure states:

- **(product-)separable states** expressed as tensor product of states of the subsystems:
  $|\psi\rangle_{AB} = |\psi\rangle_A \otimes |\psi\rangle_B = \sum_{ij} \chi_{ij} |i\rangle_A \otimes |j\rangle_B$, $\{|i\rangle_A\}, \{|j\rangle_B\}$ bases in $\mathcal{H}_A, \mathcal{H}_B$
- **entangled states** (Schrödinger 1935) nonfactorizable
  $|\psi\rangle_{AB} \neq |\psi\rangle_A \otimes |\psi\rangle_B$

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  example: $|\psi_1\rangle = \frac{1}{\sqrt{2}} |00\rangle + \frac{1}{\sqrt{2}} |01\rangle = |0\rangle_A \otimes [|0\rangle + |1\rangle]_B$
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  example: **Bell-states**

  $$|\phi^{\pm}\rangle := \frac{1}{\sqrt{2}} [|00\rangle \pm |11\rangle]$$
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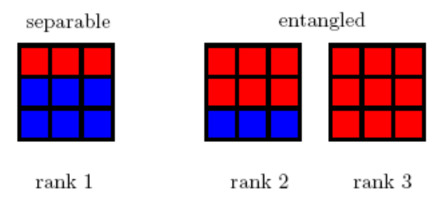
for pure states in 2-particle-systems we can easily distinguish:
Schmidt-decomposition

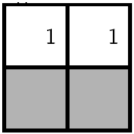
Schmidt-decomposition

- every state in $\mathcal{H}_{AB}$ can be written as:

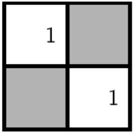
$$|\psi\rangle = \sum_{i=1}^{r} \tilde{\chi}_i \, |u_i\rangle_A \otimes |v_i\rangle_B$$

- $1 \leq r \leq \min\left(\dim(\mathcal{H}_A), \dim(\mathcal{H}_B)\right)$ and $\sum \tilde{\chi}_i^2 = 1, \; \tilde{\chi}_i$ real and positiv, $|u_i\rangle_A, |v_i\rangle_B$ are bases
- Schmidt coefficients $|\tilde{\chi}_i|^2$ are the eigenvalues of the partial traces of $\rho_{AB} = |\psi\rangle_{AB} \langle\psi|$
- **partial trace**: $\rho_B = \mathrm{Tr}_A(\rho_{AB}) = \sum_i \langle u_i| \, \rho_{AB} \, |u_i\rangle_A$
- **Schmidt-rank**: r number of nontrivial (nonzero) eigenvalues of ρ_A or ρ_B
- pure bipartite system is separable for $r = 1$





separable state $|00\rangle + |01\rangle$



Bell-state $|00\rangle + |11\rangle$

Peres-criterion

generalization of Schmidt-decomposition

- mixed composite states: hence we have to consider density matrices
- **Peres-Horodecki-criterion** (1996) aka positive partial tranpose **PPT** is a necessary criterion for ρ to be separable $(\rho = \sum_i p_i \rho_A^i \otimes \rho_B^i)$
  ρ is separable $\Longleftrightarrow \rho^{\mathrm{T_B}} := (\mathbf{1}_A \otimes \mathrm{T_B})[\rho] \geq 0$ and $\rho^{\mathrm{T_A}} \geq 0$
  $$\langle m\mu| \rho |n\nu\rangle = \langle m\nu| \rho^{\mathrm{T_B}} |n\mu\rangle$$
- for 2×2 and 2×3 this criterion is even sufficient

LOCC

A separable state is a quantum state which can be prepared in a local or classical way (LOCC): (local operations and classical communications)

or

Alice and Bob cannot turn a disentangled state into an entangled state by LOCC.

or

By LOCC alone, Alice and Bob cannot increase the total amount of entanglement which they share.

Von-Neumann-entropy

system with density matrix ρ

$$S(\rho) := -\operatorname{Tr}(\rho \log \rho)$$

representation in eigenbasis of ρ: $\log \rho = \sum_i \log \lambda_i |i\rangle \langle i|$ all eigenvalues λ_i known $\rightarrow$

$$S(\rho) = -\sum_k \lambda_k \log \lambda_k$$

Why logarithm?

- we need a monotonous function
- logarithm is additive for independent systems $S(\rho = \rho_A \otimes \rho_B) = S(\rho_A) + S(\rho_B)$

Entanglement measures for pure states

the **entanglement of a pure** quantum system $|\psi\rangle$ is :

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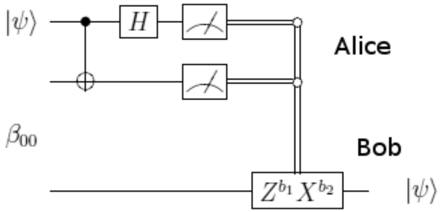
$$|\phi\rangle = \frac{1}{\sqrt{2}} \left[|00\rangle + |11\rangle \right] \Rightarrow \rho_A = \frac{1}{2}\mathbf{1} \Rightarrow S(\rho_A) = 1$$

One-Qubit Teleportation circuit



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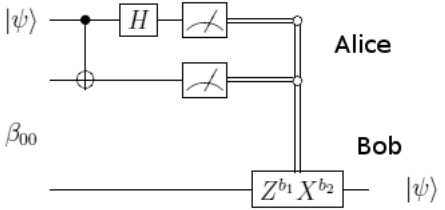

Protocoll: 1. step

- Alice and Bob: initially share a Bell state $\beta_{00} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ (maximally entangled), e.g. both have a spin-1/2-particle and move away from each other
- Alice wants to transmit $|\psi\rangle_Z = \alpha\,|0\rangle_Z + \beta\,|1\rangle_Z$
- in total: $|\psi\rangle_Z \otimes \beta_{00} =$ $\frac{1}{\sqrt{2}}\left[\alpha(|0\rangle_Z|00\rangle + |0\rangle_Z|11\rangle) + \beta(|1\rangle_Z|00\rangle + |1\rangle_Z|11\rangle)\right]$

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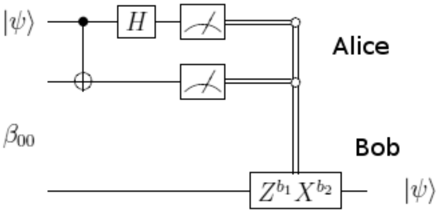

Protocoll: 2. step

Alice: joint measurement upon the qubit to be transmitted and her half of the the Bell-state (apply CNOT(1,2) and Hadamard(1)); measuring both qubits in the computational basis completes the Bell measurement

Recall: CNOT and Hadamard

- standard gates that form with the pauli-matrices and the unit an infinite set of unitary operations
- **CNOT** in computational basis representation:

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{bmatrix} \mathbf{1}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & \sigma_x \end{bmatrix}$$

- **Hadamard** in computional basis: $\text{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

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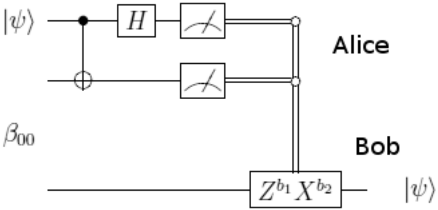
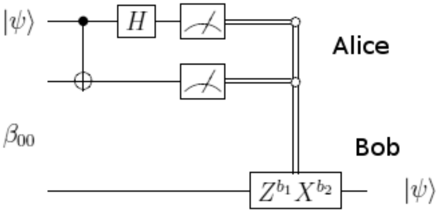


Protocoll: 2. step

$$\mathrm{CNOT}(1,2) \rightarrow \frac{1}{\sqrt{2}}\left[\alpha(|0\rangle(|00\rangle + |11\rangle) + \beta(|1\rangle(|10\rangle + |01\rangle)\right]$$

One-Qubit Teleportation circuit



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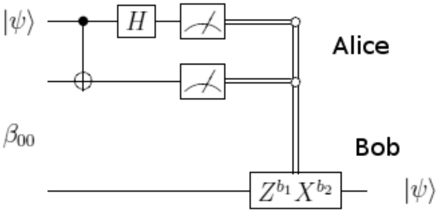
$$\begin{aligned} \text{H}(1) \quad &\rightarrow \quad \frac{1}{2}\left[\alpha(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) + \beta(|0\rangle - |1\rangle)(|10\rangle + |01\rangle)\right] \\ &= \quad \frac{1}{2}\left[|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\alpha|1\rangle + \beta|0\rangle)\right] + \\ &+ \quad \frac{1}{2}\left[|10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(\alpha|0\rangle - \beta|1\rangle)\right] \end{aligned}$$

One-Qubit Teleportation circuit



Protocoll: 2. step

$$\begin{aligned}
\text{H(1)} \quad &\rightarrow \quad \frac{1}{2}[\alpha(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) + \beta(|0\rangle - |1\rangle)(|10\rangle + |01\rangle)] \\
&= \quad \frac{1}{2}[|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\alpha|1\rangle + \beta|0\rangle)] + \\
&+ \quad \frac{1}{2}[|10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(\alpha|1\rangle - \beta|0\rangle)] \\
|\psi^{total}\rangle \quad &= \quad \frac{1}{2}\sum_{b_1,b_2=0}^{1} |b_1 b_2\rangle (X^{b_2} Z^{b_1})|\psi\rangle
\end{aligned}$$

Teleportation protocoll

Protocoll 2. step: measurement

- projection on the subspace, defined by one of the projectors $\{P_\mu\} = \{|00\rangle\langle 00|, |10\rangle\langle 10|, |01\rangle\langle 01|, |11\rangle\langle 11|\}$
- Alice will obtain one of the four basis states, with equal probability
- e.g. $|01\rangle$: then the state of 3 qubits will collapse to $|01\rangle(\alpha|1\rangle + \beta|0\rangle)$

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Protocoll 3. step:

- Alice sends her result to Bob **classicly** (2 c-bits needed)

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Protocoll 4. step:

- Bob knows exactly what to do to complete the teleportation: he has to perform an unitary transformation on his qubit

received bits	operation
00	$\mathbf{1}$
01	X
10	Z
11	ZX

Bob applies $X = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ to his qubit and completes the teleportation

Remarks on Teleportation

- **resources used?** 2 c-bit + 1 e-bit to "transport" 1 q-bit
- due to the classical transmission: operation is **not superluminal**
- there is not 2 copies of $|\psi\rangle$ at any time (no problem with the **no-cloning-theorem**)

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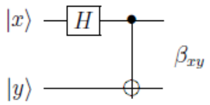
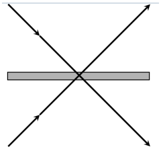
How to get the Bell-state (EPR-pair)?





e.g. input in computational basis

$|00\rangle$: $\rightarrow \mathrm{CNOT}(1,2)\,\mathrm{H}(1)\,|00\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \beta_{00}$

the Hadamard-gate acts like a beam splitter:
$\mathrm{H}\,|\psi\rangle_{in} = \mathrm{H}[\alpha\,|0\rangle_{in} + \beta\,|1\rangle_{in}] = \frac{1}{\sqrt{2}}[(\alpha+\beta)\,|0\rangle_{out} + (\alpha-\beta)\,|1\rangle_{out}]$

experiment: polarization entanglement with birefringent crystall

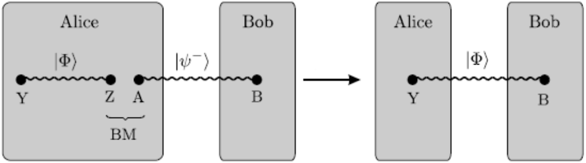
Entanglement swapping (just the idea)

idea

- Alice has: 2 entangled qubits Y and Z in state $|\phi\rangle_{YZ}$ and qubit A of an entangled state $|\psi^-\rangle_{AB}$
- Bob has: the other qbit of $|\psi^-\rangle_{AB}$

objective

- form with Y and B the state $|\phi\rangle$ (entanglement between Alice's and Bob's system)



Introduction

entanglement

Teleportation

Circuit and Protocoll

Discussing the results

Why is entanglement necessary?

- Assume we have the shared state $|00\rangle$. No matter what Alice does, Bob's qubit will always be $|0\rangle$. $\rightarrow$ no correlation.
- classical correlation is not sufficient either: $\rho_{AB} = 1/2(|00\rangle\langle 00| + |11\rangle\langle 11|)$, statistical mixture, but not entangled. clearly correlated, but there wan't be teleportation.

Entanglement purification

- Alice and Bob need a maximally entangled state for teleportation.
- non-ideal conditions: decoherene and dissipation (e.g. in a long optic fibre)
- impurities in the fibre will disturb the singlet state, after transmission Alice and BOb won't share a singlet, but a mixture of non-maximally entangled state

Hence 'amplification' or concentration needed → **entanglement purification**
purification of arbitrary states under a condition (by LOCC):
$F(\rho) = \max_{allmax.ent|\psi\rangle} \langle\psi| \rho |\psi\rangle$. purifiable e.g. for $F(\rho) > 0.5$

Entanglement measures again

- $E(\sigma) = 0$ for disentangled/separable state
- $E(\sigma) = E(U\sigma U^{\dagger})$ for any state σ and any local unitary transformation the entanglement remains unchanged
- LOCC cannot increase the expected entanglement $E(\sigma) \geq \sum_i p_i E(\sigma_i)$
- 2 pairs of entangled particles in total state $\sigma = \sigma_1 \otimes \sigma_2$: $E(\sigma) \leq E(\sigma_1) + E(\sigma_2)$
- mixture of states should not increase E: $E(\lambda\rho + (1-\lambda)\rho) \leq \lambda E(\rho) + (1-\lambda)E(\rho), 0 \leq \lambda \leq 1$

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- LOCC cannot increase the expected entanglement $E(\sigma) \geq \sum_i p_i E(\sigma_i)$
- 2 pairs of entangled particles in total state $\sigma = \sigma_1 \otimes \sigma_2$: $E(\sigma) \leq E(\sigma_1) + E(\sigma_2)$
- mixture of states should not increase E: $E(\lambda\rho + (1-\lambda)\rho) \leq \lambda E(\rho) + (1-\lambda)E(\rho), 0 \leq \lambda \leq 1$