Theory of Subpicosecond Pulse Generation by Active Modelocking of a Semiconductor Laser Amplifier in an External Cavity: Limits for the Pulsewidth

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***Abstract*—Active modelocking of a semiconductor laser amplifier (SCLA) in an external cavity is theoretically modeled, using a $\cosh^{-2}$-description for the optical pulses. Analytical solutions are derived. The model includes finite gainwidth, which is shown to be of the largest importance for a correct description, and gain saturation. A lower limit of the optical pulsewidth is derived in terms of the gainwidth, the injection current, the external loss, and other parameters. In recent experiments Bowers *et al.* found, surprisingly, pulses being a factor of 10 to 100 broader than the inverse gainwidth, and trailing pulses with an intensity almost of the same order of magnitude as the leading one even for SCLA facet reflectivities as low as 10^{-2}. Both features are quantitatively explained by our theory.**

I. Introduction

Generation of ultrashort optical pulses by various methods using semiconductor lasers is of great current interest [1]–[15] because of the potential applications in optical communication and measurement systems. As compared to gain switching, active modelocking requires more expensive and complex techniques but is superior concerning the amplitude and timing jitter and the minimum width of the obtainable pulses [1], [16]–[18]. Recent experimental results [1] show that sub-ps pulses can be obtained with this technique.

While single pulse amplification [10]–[14] and active modelocking [1], [22]–[24] has been treated numerically by a number of authors, an analytical treatment of active modelocking has been only given by [19]–[21]. The analytical solutions presented in [19] and [20] are not restricted in their applicability to any particular optical pulse shape and were derived in the frequency domain. Assuming a particular pulse shape and working in the time domain in a similar way, as done in [21], allows us to include the effects of gain saturation and to assume arbitrary injection pulses and nonsinusoidal gain curves, which is not done in [19] and [20]. Contrary to most of the numerical work [1], [23], [24] we include the finite gainwidth. We are able to show that its neglect can lead to unrealistically short pulses. Our model explains quantitatively and to our knowledge for the first time, why the lower limit of the pulsewidth imposed by the finite gainwidth is not reached and why reflections at an antireflection (AR)-coated semiconductor laser amplifier (SCLA) facet with 1% reflectivity is sufficient to produce trailing pulses still having 50% of the leading pulse intensity.

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This paper is organized as follows. In Section II our theory is evaluated "locally": the changes of the optical pulsewidth, the pulseheight, and the time at which the pulse maximum occurs, are calculated for a pulse propagation through an infinitesimally thin slice of the SCLA. The effects of a Lorentzian shape of the gain rather than a constant gain are treated in Section III, and in Section IV the integral equations for the changes in the pulse shape during one round-trip are given. Their evaluation yields the dependence of the pulsewidth upon various parameters, which is presented in Section V. The effects of multiple reflections caused by a nonzero SCLA facet reflectivity are discussed in Section VI, while in Section VII the approximation of the pulse shape by a $\cosh^{-2}$-function is justified. In Section VIII we conclude and discuss how the pulsewidth can be further decreased and how the trailing pulses can be avoided.

II. The Local Model

Our model starts with the common rate equations [25] for the carrier density $N(z, t)$

$$\frac{\partial N(z, t)}{\partial t} = \frac{\eta}{ed} J(t) - BN^2 - G_0(N - N_0)[S^+(z, t) + S^-(z, t)] \tag{1}$$

and the traveling wave equation for the photon densities $S^{\pm}(z, t)$, traveling in the positive and negative z-direction, respectively:

$$\frac{\partial S^{\pm}(z, t)}{\partial t} \pm v_g \frac{\partial S^{\pm}(z, t)}{\partial z} = \Gamma G_0(N - N_t) S^{\pm}(z, t) \tag{2a}$$

$$N_t = \frac{\kappa}{G_0 \Gamma} + N_0 \tag{2b}$$

together with the boundary conditions (Fig. 1):

$$S^+(0, t) = R_1 S^-(0, t) \tag{3a}$$

$$S^-(L, t) = R_2 S^+(L, t) + R_{\text{ext}}(1 - R_2)^2 S^+(L, t - \tau_{\text{ext}}) \tag{3b}$$

Here N is the carrier density, η is the current injection efficiency, e is the elementary charge, d is the active layer thickness, J is the injection current density, Γ is the optical confinement factor, B is the rate coefficient for the spontaneous

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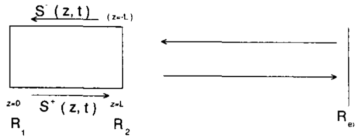

Fig. 1. Scheme of the active mode-locking configuration.

emission, N_0 is the transparency concentration. G_0 is the gain, which is at this stage of the model assumed to be constant for all optical frequencies, and κ accounts for the losses inside the SCLA (without facet reflection losses). N_t is the threshold for amplification one, i.e., the carrier density, for which an optical signal is neither amplified nor damped between the SCLA facets. $v_g = L/\tau_{\text{SCLA}}$ is the group velocity, where L is the length of the SCLA, τ_{SCLA} is the SCLA single pass time, and τ_{ext} is the time for a round-trip in the external cavity (excluding the SCLA). R_1 and R_2 are the SCLA facet reflectivities and R_{ext} is the reflectivity of the external cavity. The SCLA-cavity system is shown in Fig. 1.

The main approximations made in (1)–(3) are as follows. The first approximation is a spatially constant injection current density. Inhomogeneous injection currents can lead to regions of saturable absorption in the SCLA which would cause an additional pulse shaping. Homogeneity is assumed for the sake of simplicity and to enable an analytic solution. Thus our model is restricted to SCLA's with one electrical contact, driving the whole active area. The second is the use of photon densities rather than of amplitudes. Thus any phase relation between the internally (at the SCLA facets) or externally reflected optical field amplitude and the field inside the laser is neglected. The neglect of the phase of the internal field is justified, if the pulsewidth is significantly shorter than the single pass time, as it is the case here. The phase relation between the field reflected from the external cavity and the field inside the laser may be neglected, because in a stable pulsating mode the laser is operated below threshold between two optical pulses, i.e., almost during the whole external round-trip time. During this time, which is about one order of magnitude longer than the photon lifetime, the SCLA "forgets" the phase of the internal field due to the spontaneous emission. The third is the neglect of spontaneous emission in (2). For synchronously mode-locked dye lasers fluctuations in the pulse shape are found for incoherent spontaneous emission noise [28], [29]. In previous numerical work [22] we found stable, pulsating solutions also for a spontaneous emission factor of 10^{-4}, using broader injection current pulses and longer cavities which leads, however, to significantly longer pulses than in this work. As our theory implicitly presupposes the existence of a stable pulsating solution, the neglect of spontaneous emission is unavoidable. Our theory predicts a lower limit for the pulsewidth which can be reached for sufficiently low spontaneous emission only. The fourth includes a dispersion-free SCLA. The dispersion due to the carrier density dependence of the refractive index is very small, as the carrier depletion due to the emission of the pulse is below 3% for realistic parameters (10). For gain switching, e.g., this variation is about 10 times stronger and dispersion plays an important role. In previous work [22] we found a shift of the spectrum for a nonzero linewidth enhancement factor, but no effect on the pulsewidth. The effect of the material dispersion is assumed to be negligible due to the short length of the SCLA. For pulsewidths lower than 100 fs, however, this neglect can not be justified [30]. And the last approximation is a dispersion-free external cavity, which can be desribed by an effective reflection coefficient R_{ext}.

Introducing the normalized quantities

$$n = \frac{N}{N_t}; \quad n_0 = \frac{N_0}{N_t}; \quad s^{\pm} = \frac{S^{\pm}}{N_t};$$

$$j = \frac{\eta}{ed}\frac{J}{BN_t^2}; \quad b = BN_t; \quad g_0 = G_0 N_t;$$

which are denoted by lower case letters, (1) and (2) read

$$\frac{\partial n(z, t)}{\partial t} = b[j(t) - n^2] - g_0(n - n_0)[s^+(z, t) + s^-(z, t)] \tag{4}$$

$$\frac{\partial s^{\pm}(z, t)}{\partial t} \pm v_g \frac{\partial s^{\pm}(z, t)}{\partial z} = \Gamma g_0(n - 1)s^{\pm}(z, t). \tag{5}$$

To enable an integration of (4) and (5) let us now consider a $\cosh^{-2}$ shaped optical pulse of width τ peaked at $t = t_0$ for $z = 0$, traveling for simplicity in the positive z direction:

$$s(z, t) = \frac{s_0(z)}{\cosh^2\left(\dfrac{t'}{\tau(z)}\right)}; \quad t' = t - t_0(z) - \frac{z}{v_g}. \tag{6}$$

Here t' is the time relative to the peak of the optical pulse. To account for changes in the pulse shape $s_0(z)$, $\tau(z)$, and phase $t_0(z)$ those parameters are chosen as z dependent. To make an integration of (4) possible: 1) the injection current is expanded around $t' = 0$ and 2) a fixed value of n_{st} is assumed on the right-hand side of (4) instead of $n(z, t)$.

$$\frac{\partial n(z, t)}{\partial t} = b\left[j(t' = 0) + t'\frac{dj}{dt}(t' = 0) - n_{\text{st}}^2\right] - g_0(n_{\text{st}} - n_0)s(z, t) \tag{7}$$

The integration interval will be the "photon dominated phase," which comprises an interval of several τ's around t_0. The assumption 1) is justified for ps pulses. Assumption 2) is an approximation to the exact solution of the differential equation (4) and is justified by the result of this analysis, namely that the carrier density change due to stimulated emission is of the order of only 1%.

Now (7) can be integrated for a fixed position in the SCLA:

$$n(t') = n_{\text{st}} + bt'\left[j(t' = 0) + \frac{t'}{2}\frac{dj}{dt}(t' = 0) - n_{\text{st}}^2\right] - g_0(n_{\text{st}} - n_0)s_0(z)\tau(z)\left[\tanh\left(\frac{t'}{\tau(z)}\right) + 1\right] \tag{8}$$

n_{st} has been chosen as the starting value of the carrier density, i.e., the value of n, which an incoming pulse would see without spontaneous emission or current injection.

For later use, the carrier density changes due to current injection during the duration of a round-trip in the external cavity

$$\Delta n_j := \int_0^{\tau_{\text{ext}}} b(j(t) - n_{\text{st}}^2)\, dt \tag{9}$$

and due to stimulated emission

$$\Delta n_s(z) := -\int_0^{\tau_{ext}} g_0\,(n_{st} - n_0)[s^-(z, t)) + s^+(z, t)]\,dt$$
$$\approx -2\int_{-\infty}^{\infty} g_0(n_{st} - n_0)s(z, t)\,dt$$
$$= -4\, g_0(n_{st} - n_0)s_0(z)\tau(z) \qquad (10)$$

are defined.

With (6) and (8), (5) can be rewritten

$$\frac{ds_0}{dz} + 2s_0 \tanh\left(\frac{t'}{\tau}\right)\left(\frac{1}{\tau}\frac{dt_0}{dz} + \frac{t'}{\tau^2}\frac{d\tau}{dz}\right)$$
$$= \frac{\Gamma g_0}{v_g}\left(n_{st} - 1 + bt'\left[j(t' = 0) + \frac{t'}{2}\frac{dj}{dt}(t' = 0) - n_{st}^2\right]\right.$$
$$\left. - g_0(n_{st} - n_0)s_0(z)\tau(z)\left[\tanh\left(\frac{t'}{\tau(z)}\right) + 1\right]\right) s_0(z). \qquad (11)$$

Equation (11) is valid in a time domain of sufficiently small t' where the $\cosh^{-2}$ pulse is localized. Therefore we can expand $\tanh(t'/\tau) \approx t'/\tau$. Because (11) must hold for all allowed values of t', the terms of order t'^0, t'^1, t'^2 can be equated separately:

$$\frac{ds_0}{dz} = \frac{\Gamma g_0}{v_g}(n_{st} - 1)\left(1 - \frac{s_0}{s_{sat}}\right)s_0 \qquad (12a)$$

$$\frac{dt_0}{dz} = \frac{\Gamma g_0}{2v_g}\tau^2\,[b(j(t' = 0) - n_{st}^2) - g_0(n_{st} - n_0)s_0] \qquad (12b)$$

$$\frac{d\tau}{dz} = \frac{\Gamma g_0}{4v_g}\tau^3 b\,\frac{dj}{dt}(t' = 0) \qquad (12c)$$

with

$$s_{sat} = \frac{n_{st} - 1}{g_0(n_{st} - n_0)\tau}. \qquad (12d)$$

At this stage of the model a physical interpretation of (12) is useful. Equation (12a) describes the exponential amplification of an optical pulse in the SCLA, together with a saturation term characterized by a saturation intensity s_{sat}. This saturation is an effect of the gain depletion, and it guarantees stability of the pulse height against random fluctuations.

Equation (12b) describes the result that the pulse can travel through the SCLA faster or slower than with its group velocity which is at first sight astonishing. This deviation is a consequence of the gain saturation: an amplification in a depletable medium shows a larger amplification for the leading edge of a pulse than for the trailing one, which is equivalent to a shift of the pulse maximum towards earlier times, described by (12b). Counteracting force is the injection current, which reduces depletion. To answer the question of the stability of the peak time t_0, consider an optical pulse in a stable ringing system with a random fluctuation towards later times. If the injection current increases during the time of pulse amplification the (randomly) delayed pulse sees a higher injection current and thus is delayed still more. So on the rising shoulder of the injection current no stable pulse amplification is possible in an external cavity.

Equation (12c) describes the changes in the pulsewidth and can be interpreted in a similar way as (12b). Suppose that j, s_0, and n_{st} have values which lead to $dt_0/dz = 0$. Then (for a decreasing injection current) the leading edge of the pulse sees a higher injection current and is therefore shifted backwards, while the trailing edge sees a lower injection current and is shifted forward. This is equivalent to pulse shortening. Note that according to (8c) for a negative injection current slope the pulsewidth always will tend to zero. The counteracting force, the finite gainwidth, will be introduced in the next chapter removing this effect.

All described features are in principle valid for pulses of an arbitrary shape. Our analysis here is restricted to $\cosh^{-2}$ pulses, because this is to our knowledge the only shape which remains invariant under a propagation as described by (4) and (5) and for which analytic results can be obtained.

III. The Gain Linewidth

As a pulse broadening term in (12c), the finite gain linewidth must be included. It will be done by amplifying the optical pulse in the frequency domain with the Lorentzian gain profile

$$g(\omega) = \frac{g_0}{1 + \left[\frac{\omega}{\omega_g}\right]^2}. \qquad (13)$$

It will be shown later that it is sufficient to confine attention to a z-independent ($t_0 = 0$) $\cosh^{-2}$-pulse

$$f(t) := \frac{s_0}{\cosh^2\left(\frac{t}{\tau}\right)} \qquad (14a)$$

with its Fourier transform $F(\omega)$,

$$F(\omega) = \frac{1}{2}\frac{\omega\pi\tau^2}{\sinh\left(\frac{\omega\pi\tau}{2}\right)}s_0. \qquad (14b)$$

Any changes dF in $F(\omega)$ can be interpreted either as a result of changes $d\tau$, ds_0 in τ, s_0

$$\frac{dF}{F} = \left(2 - \frac{y}{\tanh(y)}\right)\frac{d\tau}{\tau} + \frac{ds_0}{s_0}; \qquad y := \frac{\pi\omega\tau}{2} \qquad (15)$$

or as a result of an amplification with $g_{eff} \cdot dz$,

$$\frac{dF}{F} = \frac{g_{eff}}{1 + \left[\frac{\omega}{\omega_g}\right]^2}dz. \qquad (16)$$

Expanding

$$\frac{y}{\tanh(y)} \approx \frac{1}{1 - \frac{y^2}{3} + O(y^4)}$$

gives an error of less than 1% for $\tau < 1/\pi\omega$. The results of this analysis will show that this condition holds very well for experimentally reasonable parameters. Equating (15) and (16) under this approximation yields two relations for the changes in s_0 and τ due to the Lorentzian gain profile, when the first four orders of ω are retained:

$$\frac{d\tau}{dz} = \frac{1}{\tau}\frac{12}{(\pi\omega_g)^2}g_{eff} \qquad (17a)$$

$$\frac{ds_0}{dz} = \left[1 - \frac{12}{(\pi\tau\omega_g)^2}\right]s_0 g_{eff}. \qquad (17b)$$

TABLE I
PARAMETERS FOR THE CALCULATIONS

Variable	Symbol	Value	Unit
differential gain	G_0	$1.8 \cdot 10^{-6}$	cm^3/s
spontaneous emission coefficient	B	$1.6 \cdot 10^{-10}$	cm^3/s
optical confinement factor	Γ	0.3	
carrier density for transparency	N_0	$1.2 \cdot 10^{18}$	cm^3
group velocity	v_g	$0.75 \cdot 10^8$	m/s
gain linewidth	ω_g	100	1/ps
distributed loss	κ	0.5	1/ps
SCLA length	L	300	μm
SCLA transit time	τ_{SCLA}	4	ps
SCLA facet reflectivities	R_1, R_2	1, 0	
reflectivity of the external cavity	R_{ext}	0.4	
external cavity round-trip time	τ_{ext}	64	ps
injection current period	τ_J	72	ps

Neglecting the details of the amplification we set $g_{\text{eff}} := (\ln V_{\text{sat}})/(2L)$ where V_{sat} is the saturated or effective amplification for one round-trip, which must (for stable operation) equal the external cavity loss and the SCLA facet transmission loss

$$V_{\text{sat}} = \frac{1}{(1 - R_2)^2 R_{\text{ext}}}.$$

For typical parameters ($\tau = 1$ ps, $\omega_g = 50 \cdot 2\pi/\tau_{\text{SCLA}}$, $V_{\text{sat}} = 5$, $\tau_{\text{SCLA}} = 4$ ps, $L = 300\mu$) one obtains $\Delta\tau := 2L \cdot d\tau/dz = 0.005\tau$ and $12/(\pi\tau\omega_g)^2 = 0.0002$, which justifies the neglect of the second term in (17b). As a result, (18) contains together the pulse compression (12c) and the pulse broadening (17a):

$$\frac{d\tau}{dz} = \frac{\Gamma g_0}{4v_g}\tau^3 b\frac{dj}{dt}(t' = 0) + \frac{1}{\tau}\frac{12}{(\pi\omega_g)^2}\frac{\ln V_{\text{sat}}}{2L}. \tag{18}$$

In (18) dj/dt is the only term, which can vary significantly during one round-trip of the pulse through the SCLA. With an averaged

$$j_{\text{av}}(t) := \frac{1}{2L}\int_{-L}^{L} j(t - z/v_g)\, dz \tag{19}$$

the integration of (18) and the steady-state condition

$$0 = \int_{-L}^{L}\frac{d\tau}{dz}\, dz$$

yield $\left(\text{for} \quad \frac{dj_{\text{av}}(t)}{dt}(t = t_0) < 0\right)$

$$\tau = \left[\frac{24\,(\ln V_{\text{sat}})\, v_g}{[\pi\omega_g]^2\Gamma g_0 Lb\left|\frac{dj_{\text{av}}}{dt}(t = t_0)\right|}\right]^{1/4} \tag{20}$$

For typical parameters (Table I) and $dj_{\text{av}}/dt = -1$/ps one obtains a pulsewidth of $\tau = 1$ ps connected with a time-gainwidth product of $\tau\omega_g = 2\pi \cdot 16$.

Equation (20) is one of the main results of this paper. It exhibits the usual inverse square-root dependence of the pulsewidth on the gainwidth [20], [21] and the inverse fourth-root dependence on the modulation strength dj_{av}/dt, which is also found in more general approaches [19], [21]. Equation (20) explains, to our knowledge for the first time, why the pulses produced by active modelocking are far away from being Fourier-transform limited with respect to the spectral gain linewidth [1], [16]–[18], [22]–[24]. It is evident from (18) that a complete theory of active modelocking must contain both the pulse shaping mechanism, which is included in the common traveling wave equation, and the pulse broadening, which requires an inclusion of the spectral gain dependence [22].

IV. THE INTEGRATED EQUATIONS

For a complete analysis of the changes in τ, t_0, s_0 for one round-trip through the SCLA (12a), (12b), and (18) have to be integrated. The integration of (18) is already performed in (20). The small change in τ for one round-trip justifies the assumption of τ = constant for the integration of (12a) and (12b). Furthermore, for simplicity $R_1 = 1$ and $R_2 = 0$ is assumed and the depletion of carriers by the left traveling pulse is neglected in the amplification, when the pulse travels to the right through the already depleted medium.

So (12a) and (12b) can be integrated from $z = -L$ to $z = +L$ (Fig. 1). With the unsaturated amplification V:

$$V = \exp\left[\Gamma g_0(n_{\text{st}} - 1)\frac{2L}{v_g}\right] \tag{21}$$

the saturation intensity S_{sat} defined in (12d), and the saturated amplification V_{sat}:

$$V_{\text{sat}} = \frac{V}{1 + (V - 1)\dfrac{s_{0,\text{in}}}{s_{\text{sat}}}} \tag{22}$$

one obtains

$$s_{0,\text{out}} = V_{\text{sat}} s_{0,\text{in}} \tag{23a}$$

$$t_{0,\text{out}} - t_{0,\text{in}} = \frac{\pi^2\Gamma g_0 L}{v_g}\left[b(j_{\text{av}}(t_0 - n_{\text{st}}^2) - g_0(n_{\text{st}} - n_0)s_{\text{sat}}\left(1 - \frac{\ln V_{\text{sat}}}{\ln V}\right)\right] \tag{23b}$$

where the subscripts "in" ("out") denote the values for the incoming (outcoming) pulses, i.e., at $z = -L$ and $z = L$, respectively. In (23b) it has been assumed that t_0 is approximated by an average value on the RHS. This is justified by the result below that t_0 varies only little.

Additionally, the total carrier density change Δn for one round-trip in the external cavity

$$\Delta n := \Delta n_J + \Delta n_S$$

can be calculated by an integration of (9) and (10), if s_0 is replaced in (10) by its average value

$$\frac{1}{2L}\int_{-L}^{L} s_0 dz = s_{\text{sat}}\left(1 - \frac{\ln V_{\text{sat}}}{\ln V}\right)$$

and is found to be

$$\Delta n = b[j_{\text{int}} - n_{\text{st}}^2\tau_{\text{ext}}] - \frac{2v_g}{\Gamma g_0 L}\ln\left[\frac{V}{V_{\text{Sat}}}\right] \tag{23c}$$

with

$$j_{\text{int}} = \int_0^{\tau_{\text{ext}}} j(t)\, dt.$$

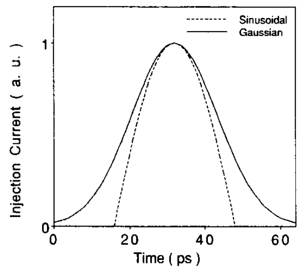

Fig. 2. Plot of the two injection currents used for the calculations. The injection current pulses are repeted with the injection current period τ_J.

The necessary conditions for steady-state pulsations are

$$V_{sat} = \frac{1}{(1 - R_2)^2 R_{ext}} \tag{24a}$$

$$j_{av}(t_0) = \frac{g_0}{b}(n_{st} - n_0)s_{sat}\left(1 - \frac{\ln V_{sat}}{\ln V}\right) + n_{st}^2 \tag{24b}$$

$$b[j_{int} - n_{st}^2 \tau_{ext}] = \frac{2v_g}{\Gamma g_0 L} \ln\left[\frac{V}{V_{sat}}\right]. \tag{24c}$$

Together with (20) these are four equations for the four unknown quantities $s_{0,in}$, τ, n_{st}, and t_0. For a given injection current a solution of (20) and (24) can be found numerically. Note that our analysis is restricted to the case of synchronous pumping ($\tau_J = \tau_{ext} + 2\tau_{SCLA}$), although asynchronous pumping could easily be introduced by adding an appropriate time delay to the left-hand side of (23b). Numerical solutions, however, show that for asynchronous pumping a stable pulsating solution often does not exist [28], thus invalidating the applicability of our theory.

V. Numerical Results

To become specific, the injection current is chosen as the positive portion of a sine-wave or as a Gaussian of comparable width (Fig. 2), with J_{max} as the only free parameter. Fig. 3 shows the calculated dependence of the pulse FWHM [FWHM = 1.763τ, Fig. 3(a)], the pulse height $s_{0,out}$ [Fig. 3(b)] and t_0 [Fig. 3(c)] upon J_{max}. For increasing injection current maximum the pulse is emitted earlier, has a higher maximum value and becomes broader. While the first two features do not depend on the special injection current shape and result from the increase in the total number of injected carriers the pulse broadening is different for both injection current shapes. This becomes evident in the τ versus t_0 plot for the same J_{max} variation. The minimum in τ roughly coincides with that value of t_0, which minimizes the (negative) slope of the averaged injection current $j_{av}(t)$. The applicable injection current is limited by a minimum level defined by the condition $\Delta n_J = 0$ and by a maximum set by the maximum number of carriers, which can be depleted by an optical pulse with a given width and a height, which is limited by the onset of gain saturation. For a nonzero SCLA facet reflectivity R_2 higher injection currents are applicable due to the occurrence of pulse trains rather than single pulses.

Fig. 4 shows the dependence of the pulsewidth upon gain linewidth. The inverse square-root behavior of (20) is also reproduced very well in the complete solution for a given injection current. To compare our results with experimental findings we estimate a square root gain-bandwidth product of $\sqrt{GB}$ =

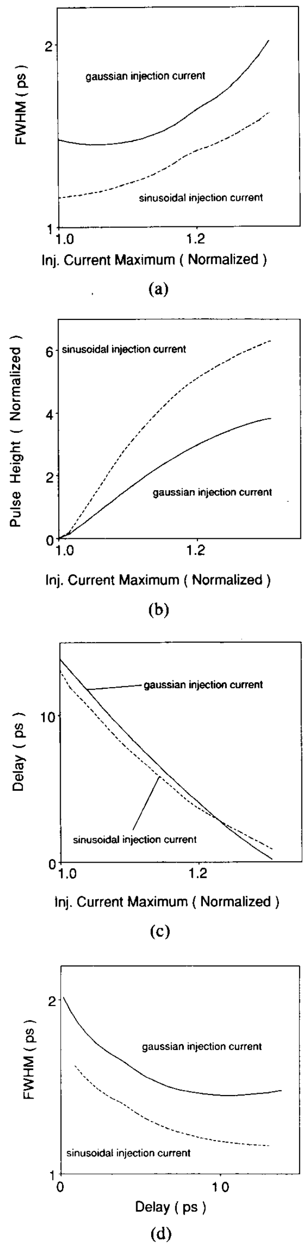

Fig. 3. Dependence of the pulse shape upon the pump strength. Equations (20) and (24) are solved for the two injection currents shown in Fig. 2. The injection current maximum is normalized to the value which is necessary to balance the spontaneous emission. (a) Pulsewidth (FWHM) versus injection current maximum, (b) pulseheight $s_{0,out}$ versus injection current maximum, (c) delay between the optical pulse and the injection current maximum, (d) pulsewidth (FWHM) versus delay time.

20 THz [31]. This corresponds to a choice of $\omega_g = 100\,\pi/\tau_{SCLA}$ in our parameters. The resulting FWHM of 1.2 ps (Fig. 4) is slightly shorter than measured with a monolithic integrated active modelocking device (1.4 ps) [27] and twice as long as obtained in an external cavity configuration (0.6 ps) [1]. This

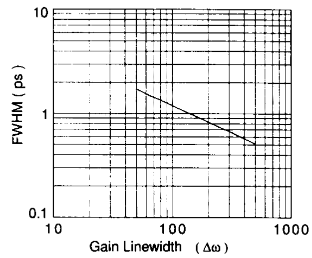

Fig. 4. Pulsewidth versus gain linewidth. The gain linewidth is plotted in units of $\Delta\omega = \pi/\tau_{SCLA}$, the different frequency of the SCLA Fabry-Perot modes.

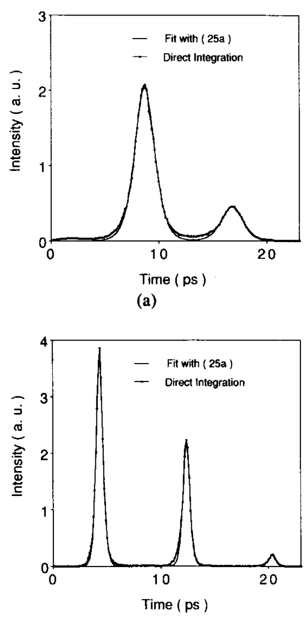

Fig. 5. Test of the $\cosh^{-2}$-approximation. Equations (1) and (2) are numerically integrated with $R_2 = 10^{-4}$ for two different values of the gainwidth ω_g and compared to the best fit obtainable with (25a). (a) $\omega_g = 60\pi/\tau_{SCLA}$, (b) $\omega_g = 120\pi/\tau_{SCLA}$.

deviation might be caused by additional pulse shortening effects in the laser (saturable absorption in aged regions of the SCLA).

VI. Multiple Reflections

Up to this stage of the model R_2 was set equal to zero to exclude multiple reflections. Experimental results [1] show that a nonzero R_2, even as low as $10^{-4} \cdots 10^{-2}$, leads to trailing pulses with a time delay of 2 τ_{SCLA} and a magnitude of about 50% of the leading pulse. This was confirmed by numerical calculations [1]. We also performed a direct numerical integration of the traveling wave equation and obtained the pulse trains shown in Fig. 5(a) and (b) with the parameters listed in Table I (except $R_2 = 10^{-4}$) and with an injection current optimized for shortest pulsewidth. The Lorentzian gain shape was included in this integration by a convolution with $\exp(-\omega_g t)$. Details of the integration procedure will be elaborated in a separate paper. Contrary to the simulations in [1], [23], and [24], which did not include the finite gainwidth we found a dependence of the pulsewidth on the gainwidth.

The nonzero R_2 can be included in this analysis by assuming a pulse train

$$s(z, t) := \sum_i \frac{s_{0,i}(z)}{\cosh^2\left(\dfrac{t_i'}{\tau_i(z)}\right)}; \tag{25a}$$

$$t_i' := t - t_{0,i}(z) - \frac{z}{v_g}$$

$$t_{0,i} = t_{0,i-1} + 2\tau_{SCLA} + \Delta t_{0,i}; \qquad \Delta t_{0,i} \ll \tau_{SCLA}$$

$$\tau_i = \tau_0 + \Delta\tau_i; \qquad \Delta\tau_i \ll \tau_0 \tag{25b}$$

rather than a single pulse. Although a complete evaluation will be given elsewhere, the question, why such small values of R_2 lead to trailing pulses of the same order of magnitude as the leading one, can be identified at this point as a question of stability. Assume that after the emission of the first pulse the SCLA is pumped sufficiently to reach the carrier density which is necessary for equality of unsaturated gain and external losses. Then (24) is valid again, but with a slightly changed value of V_{sat}

$$V_{sat,1} = \frac{V_{sat}}{1 + R_{ext}\dfrac{s_{0,0}}{s_{0,1}}}. \tag{26}$$

Thus the heights of the trailing pulses are defined by nearly the same stability condition as the first one. From (12b) with $dt_0/dz = 0$ follows that in good approximation the height of any stable pulse is proportional to $j_{av}(t_0) - n_{st}^2$. Hence the height of the trailing pulses reflects the temporal evolution of the injection current, rather than the magnitude of the internal SCLA facet reflectivity R_2.

TABLE II
FIT ERRORS FOR VARIOUS FIT FUNCTIONS[a]

Fit Function	Gaussian	$\cosh^{-2}$	$\cosh^{-1}$
data from Fig. 5(a); Δ =	13%	8%	6%
data from Fig. 5(b); Δ =	14%	9%	7%

[a]The results $d(t)$ of the direct integration plotted in Fig. 5 are fitted with three different fit functions $f(t)$. The error Δ is defined as:

$$\Delta = \frac{\int |f(t) - d(t)|\, dt}{\int |d(t)|\, dt}$$

VII. How Good Is the $\cosh^{-2}$ Approximation?

To test the validity of the $\cosh^{-2}$ approximation the pulses resulting from a direct integration of (1)–(3) (Fig. 5) are compared to the best fit obtainable with a fit function consisting of $\cosh^{-2}$ pulses (25a). The fit error for both cases using two other fit functions is presented in Table II.

For both numerical curves the $\cosh^{-1}$ fit would be apparently the best. This is in accordance with experimental results [1]. Unfortunately, assumption of $\cosh^{-1}$ pulses does not allow an analytic solution. Fig. 5 shows, however, that the numerical result can be fitted very well by a superposition of two or, respectively, three $\cosh^{-2}$ pulses.

VIII. Conclusion

Our theory gives analytic expressions for the pulse broadening and pulse shortening mechanisms as a function of the pumping strength, its derivation with respect to the time, the gainwidth and other parameters. It is shown that inclusion of both mechanisms is necessary to obtain finite pulsewidths. A tuning of the pulsewidth towards smaller values is difficult, because all parameters, except the spectral gain linewidth, influence the pulsewidth only proportionally to their fourth root. The pulsewidth can be lowered by decreasing the external losses and by decreasing the fall time of the injection current pulse. The height of the emitted pulses is mainly determined by the amplitude of the injection current at the time of their emission. So the emission of pulse trains, rather than single pulses, resulting from nonzero facet reflectivities, might disappear for injection current pulses with a fall time significantly lower than the SCLA round-trip time, which is presently unrealistic. Other possibilities to avoid the trailing pulses are the use of lasers with tilted facets [26] or the monolithic integration of the whole cavity [27].

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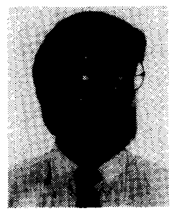

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