Noise-induced oscillations and their control in semiconductor superlattices.

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Abstract

We consider noise-induced charge density dynamics in a semiconductor superlattice. The parameters are fixed in the regime below the Hopf bifurcation that gives birth to spatio-temporal oscillations, where in the absence of noise the system rests in a fixed point. It is shown that in this case noise can induce in the superlattice quite coherent oscillations of the current through the device. While the regularity of these oscillations depends on the noise intensity, their dominant frequency remains almost constant with variation of the noise level in the system. Further, we demonstrate that a time-delayed feedback scheme that was previously used to control purely *temporal oscillations* induced by noise, can not only enhance or deteriorate the regularity of stochastic *spatio-temporal patterns* but also allows for the manipulation of the system's time scales with varying time delay.

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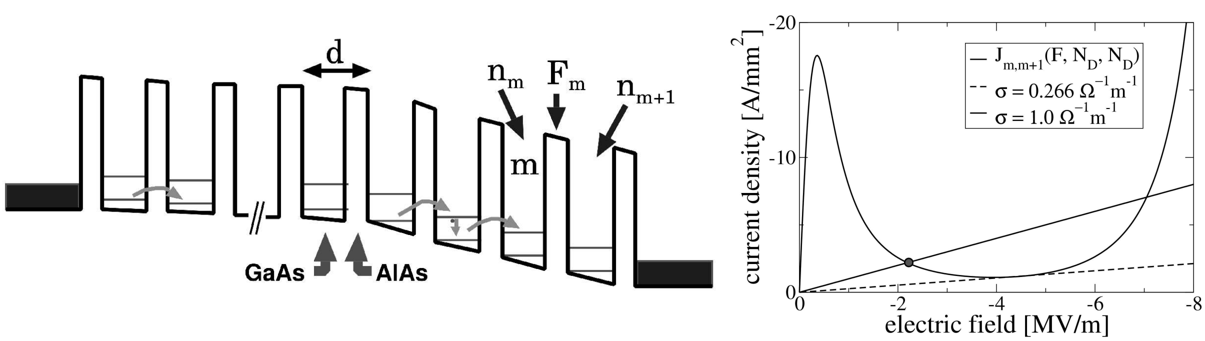
1. INTRODUCTION

Semiconductor nanostructures represent prominent examples of nonlinear dynamic systems which exhibit a variety of complex spatio-temporal patterns [Schöll, 2001]. A superlattice is such a nanostructure, which consists of alternating layers of two semiconductor materials with different band gaps. This leads to (periodic) spatial modulations of the conduction and valence band of the material, and thus forms an energy band scheme consisting of a periodic sequence of potential barriers and quantum wells (see Fig. 1(a)). Those structures can be taylored by modern epitaxial growth technologies with high precision on a nanometer scale. If the potential barriers are sufficiently thick, the electrons are localized in the individual quantum wells. In such a situation the superlattice can be treated as a series of weakly coupled quantum wells, and sequential resonant tunneling of electrons between different wells leads to strongly nonlinear charge transport phenomena, if a dc voltage is applied across the superlattice [Bonilla, 2002; Schöll, 2005; Wacker, 2002]. For instance, negative differential conductance can appear [Esaki & Tsu, 1970] (see Fig.1(b)). Thus, semiconductor superlattices can be used as generators of current oscillations, whose frequency depends on the parameters of the superlattice structure and the applied voltage, and thus, can be varied in a wide range from some hundred kHz [Cadiou *et al.*, 1994; Hofbeck *et al.*, 1996; Kastrup *et al.*, 1995; Wang *et al.*, 2000] to hundreds of GHz [Schomburg *et al.*, 1999], which makes this system very promising for practical applications. On the other hand, the inherent nonlinearity gives rise to complex spatio-temporal dynamics of the charge density and the field distribution within the device, including the formation of travelling charge accumulation and depletion fronts and field domains associated with current oscillations. Even chaotic scenarios have been found experimentally [Luo *et al.*, 1998; Zhang *et al.*, 1996] and described theoretically in periodically driven [Bulashenko & Bonilla, 1995] as well as in undriven superlattices [Amann *et al.*, 2002]. The interaction between multiple moving fronts may lead to sophisticated self-organized patterns, which are typical of a large variety of spatially extended systems [Amann *et al.*, 2003; Kapral & Showalter, 1995; Scott, 2004].

It is well known that microscopic random fluctuations essentially affect the transport mechanisms in semiconductor nanostructures [Blanter & Büttiker, 2000; Kießlich *et al.*, 2003; Song *et al.*, 2003; Zhao & Hone, 2000]. They usually smear out and deteriorate the regularity in charge transport. However, nowadays for a large class of extended systems of reaction–diffusion type it has been shown that noise can play a constructive role inducing quite coherent dynamical space-time patterns [García-Ojalvo *et al.*, 1993]. Recently, such noise-induced patterns were also found in semiconductor nanostructures described by a reaction-diffusion model for the current density distribution [Stegemann *et al.*, 2005]. Thus, the open question to what extent the noise-induced ordering occurs generally in different classes of nanostructures, becomes of central importance. Another essential issue in all those systems is the question how one can deliberately influence and control the regularity of such noise-induced dynamics.

It was recently shown for two general classes of simple nonlinear systems with temporal degrees of freedom only, that the coherence properties and the time scales of noise-induced oscillations can be changed by applying a time-delayed feedback [Balanov *et al.*, 2004; Janson *et al.*, 2004; Schöll *et al.*, 2005] in the form which was introduced earlier by Pyragas [Pyragas, 1992] for chaos control of deterministic dynamics. In that previous work purely temporal noise-induced dynamics was considered within the example of a Van-der-Pol oscillator, i.e. a system close to but below a Hopf bifurcation, and a FitzHugh-Nagumo-model, i.e. an excitable system.



(a) (b)

FIG. 1 (a) Superlattice energy band structure of alternating GaAs and AlAs layers under bias. (b) Current density vs electric field characteristic at the emitter barrier (straight line) and between two neutral wells exhibiting negative differential conductivity.

In the present paper we consider a semiconductor superlattice, i.e. a spatially extended system which in some range of parameters displays complicated front dynamics. It exhibits spatio-temporal patterns markedly distinct from those of a reaction-diffusion system

[Stegemann *et al.*, 2005]. We study the effects of noise in the superlattice for parameters fixed below the Hopf bifurcation giving birth to spatio-temporal front oscillations, where there are no self-oscillations in the deterministic system, and the only fixed point is a stationary charge depletion front. We show that in this case within a certain range of noise intensities highly coherent behaviour of the electron density dynamics arises, resulting in oscillations of the current, whose power spectrum shows very pronounced peaks. We also demonstrate that global feedback in the form of the difference between the current through the device at time t and the current at a delayed time $t-\tau$ can be used for the effective control of essential features of such *noise-induced* oscillations like time scales and coherence. This complements previous work on time-delayed feedback control of *deterministic chaos* in superlattices [Schlesner *et al.*, 2003] and in other spatially extended semiconductor nanostructures modelled by reaction-diffusion systems [Baba *et al.*, 2002; Beck *et al.*, 2002; Franceschini *et al.*, 1999; Schöll, 2004; Unkelbach *et al.*, 2003].

The paper has the following structure. After the Introduction, in Sec. II we describe our stochastic model of a semiconductor superlattice. In Sec. III noise-induced patterns in the system are discussed, and quantities characterizing such kind of dynamics are introduced. A time-delayed feedback scheme for control of noise-induced dynamics is proposed in Sec. IV, and in Sec V we discuss the obtained results and possible applications.

2. MODEL EQUATIONS.

Our model of a superlattice is based on sequential tunneling of electrons [Wacker, 2002]. The resulting tunneling current density $J_{m\to m+1}(F_m, n_m, n_{m+1})$ from well m to well $m+1$ depends only on the electric field F_m between both wells and the electron densities n_m and n_{m+1} in the respective wells (in units of cm^{-2}). For details of the microscopic calculation of $J_{m\to m+1}$ we refer to the literature [Amann *et al.*, 2001; Wacker, 2002]. A typical dependence of $J_{m\to m+1}$ on the electric field between two consecutive wells is N-shaped and exhibits a pronounced regime of negative differential conductivity, as shown in Fig.1(b).

In the following we will adopt the densities of electrons in each well as the dynamic variables of the system. The dynamic equations are then given by the continuity equation

$$e\frac{\mathrm{d}n_m}{\mathrm{d}t} = J_{m-1\to m} - J_{m\to m+1} \quad \text{for } m = 1, \dots N \tag{1}$$

where N is the number of wells in the superlattice.

The electron densities and the electric fields are coupled by the following discrete version of Gauss's law

$$\epsilon_r \epsilon_0 (F_m - F_{m-1}) = e(n_m - N_D) \quad \text{for } m = 1, \dots N, \tag{2}$$

where ϵ_r and ϵ_0 are the relative and absolute permittivities, $e < 0$ is the electron charge, N_D is the donor density, and F_0 and F_N are the fields at the emitter and collector barrier, respectively.

The applied voltage between emitter and collector gives rise to a global constraint

$$U = -\sum_{m=0}^{N} F_m d, \tag{3}$$

where d is the superlattice period.

The current densities at the contacts are crucial for the generation of front patterns injected from the emitter. For our purpose it is sufficient to choose Ohmic boundary conditions:

$$J_{0\to 1} = \sigma F_0 \tag{4}$$

$$J_{N\to N+1} = \sigma F_N \frac{n_N}{N_D} \tag{5}$$

where σ is the Ohmic contact conductivity, and the factor n_N/N_D is introduced in order to avoid negative electron densities at the collector.

Now we extend the deterministic model to incorporate stochastic influences. The dominant noise source, which effects the electron dynamics in semiconductor nanostructures, is shot noise, which is associated with the fluctuations of the times between tunneling of electrons across a potential barrier (see e.g. [Pouyet & Brown, 2003] for a theoretical description). In the case of a weakly coupled superlattice, the random component of the well-to-well current can be described in a first approximation by Poissonian statistics [Blanter & Büttiker, 2000]. Those fluctuations affect the current densities $J_{m\to m+1}$. Assuming that the tunneling times are much smaller than any characteristic time scale of the *global current* through the device $J = \frac{1}{N+1}\sum_m J_{m\to m+1}$, and taking into account that each current density $J_{m\to m+1}$ is influenced by many Poissonian events we can roughly approximate those fluctuations by Gaussian white noise sources in the continuity equations for the electron densities. Charge conservation is automatically guaranteed by adding a noise term ξ_m to each current density

$J_{m-1\to m}$:

$$e\frac{dn_m}{dt} = J_{m-1\to m} + D\xi_m(t) - J_{m\to m+1} - D\xi_{m+1}(t), \tag{6}$$

where $\xi_m(t)$ is Gaussian white noise with

$$\langle \xi_m(t) \rangle \ = \ 0, \tag{7}$$

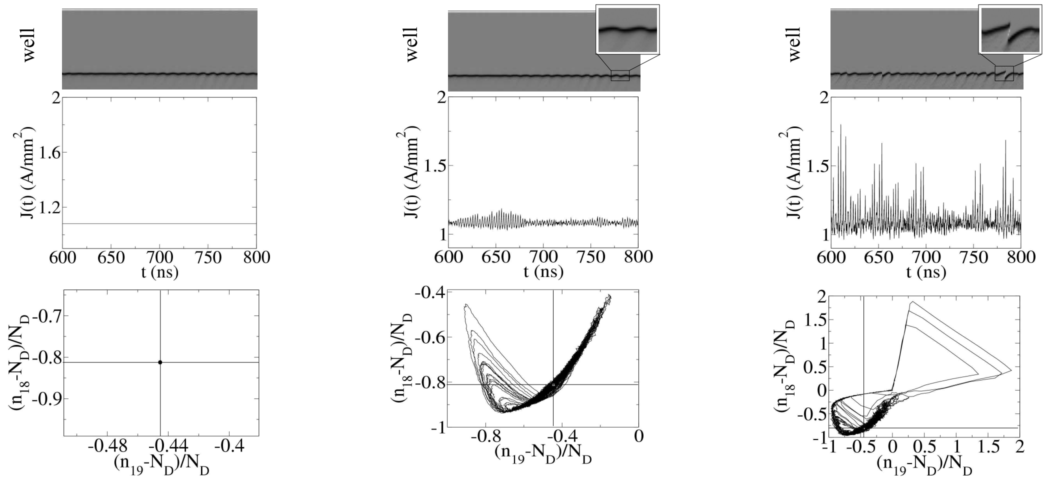
$$\langle \xi_m(t)\xi_{m'}(t') \rangle \ = \ \delta(t-t')\delta_{mm'}, \tag{8}$$

and D is the noise intensity. Since we assume that the inter-well coupling in our superlattice is very weak, these noise sources can be treated as independent.

3. NOISE-INDUCED SPACE-TIME PATTERNS

We now fix the parameters of the system slightly below a Hopf bifurcation, where without noise ($D = 0$) the only stationary solution is a stable fixed point that corresponds to a stationary depletion front localized over a small range of wells near the emitter (Fig.2(a)). This is done by choosing a very small σ which pins a high field domain at the emitter region and suppresses the generation of accumulation fronts at the emitter. Note that for the considered superlattice a free depletion front under fixed current conditions would always have a positive velocity [Amann *et al.*, 2002]. The observed stationarity of the depletion front is therefore a consequence of the global coupling (3) and the suppression of new fronts at the emitter.

As the noise intensity increases ($D > 0$), the current density starts to oscillate in a quite regular manner around the steady state (middle panel of Fig. 2(b)). From the corresponding charge density plot (upper panel of Fig. 2(b), see inset) we can associate this oscillation with a periodic motion of the depletion front as a whole. This is the expected behavior close to a Hopf bifurcation. At even larger noise intensities, however, the nature of the observed dynamics changes dramatically (Fig. 2(c)). Now the current oscillations are no longer harmonic around the stationary value, but become sharply peaked and spiky, and the average current is shifted towards larger values. This is reflected in a more asymmetric motion of the depletion front (upper panel of Fig. 2(c), see inset). In particular we now occasionally observe the onset of a tripole oscillation, where in addition to the existing depletion front, a dipole of an accumulation and a depletion front is generated close to the emitter, and the leading (but not fully developed) accumulation front catches up and annihilates with the already present depletion front, while the trailing depletion front remains.



(a) (b) (c)

FIG. 2 Dynamical behavior of the superlattice for different noise intensities (a) $D = 0$ (b) $D = 0.1As^{1/2}/mm^2$ and (c) $D = 0.5As^{1/2}/mm^2$ for fixed $\sigma = 0.266\Omega^{-1}m^{-1}$ and $U = 1V$ (Other parameters as in [Schlesner *et al.*, 2003]). The upper panel shows space-time plots of the electron density (black color indicates electron depletion, light color electron accumulation; the emitter is at the bottom. The insets depict a blow-up of the depletion front). The middle panel shows the total current density vs. time. The lower panels display the corresponding phase portraits of electron densities n_{18} vs. n_{19} in two wells close to the emitter.

To quantify the regularity of oscillations we introduce the correlation time t_{cor} given by the formula [Stratonovich, 1963] :

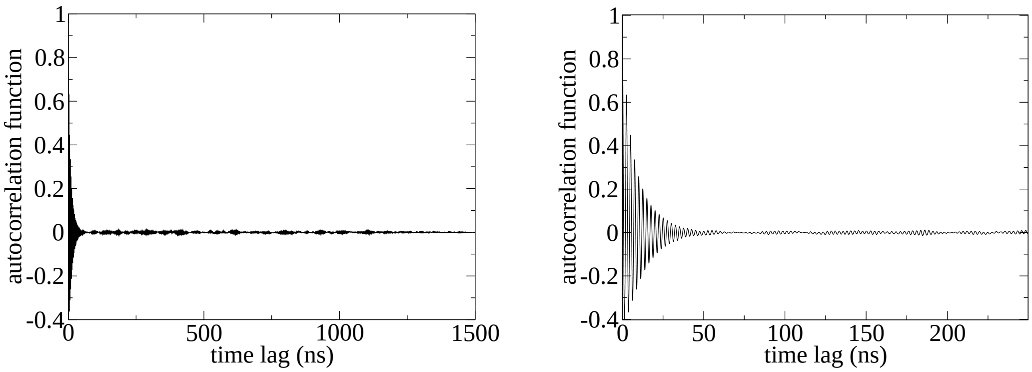
$$t_{cor} = \frac{1}{\psi(0)} \int_0^\infty |\psi(s)| ds, \tag{9}$$

where $\psi(s)$ is the autocorrelation function of the current density signal $J(t)$,

$$\psi(s) = \langle (J(t) - \langle J \rangle)(J(t-s) - \langle J \rangle) \rangle, \tag{10}$$

and $\psi(0)$ is its variance. Note that in (10) we apply an ensemble average.

A typical numerical estimate of the autocorrelation function for the noise–induced oscillations of our superlattice is shown in Fig. 3. Qualitatively we see an exponentially damped



(a) (b)

FIG. 3 Numerical estimate of the autocorrelation function of noisy ($D = 0.5As^{1/2}/mm^2$) current density time series: complete data (a) and enlarged part for small time lag (b).

oscillation which can be approximated by

$$\psi(s) = \psi(0)\exp(-s/t_e)\cos(\omega_0 s), \tag{11}$$

with a noisy tail, which will vanish with increasing size of the statistical ensemble. By comparison with (9) we find for $\omega_0 t_e \gg 1$ the relation $t_e \approx (\pi/2)t_{cor}$ [Schöll *et al.*, 2005].

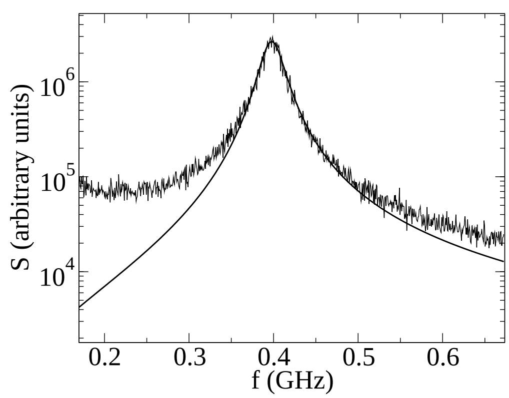
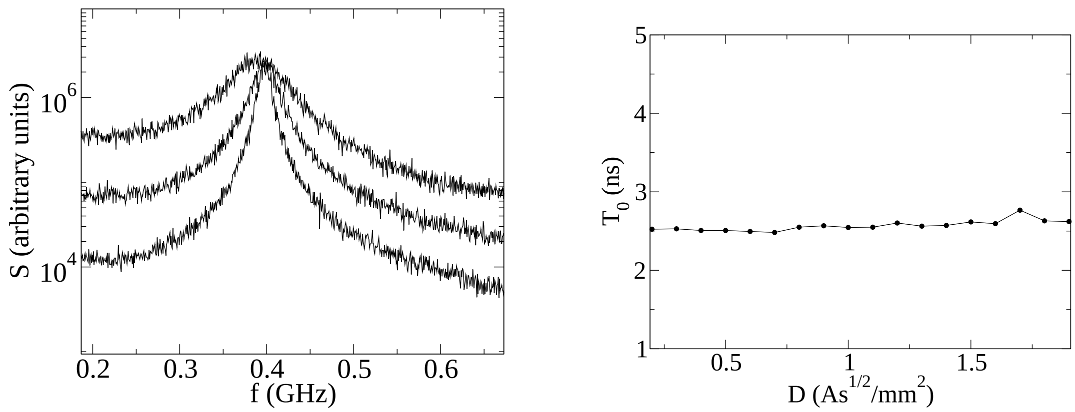


FIG. 4 Main spectral peak for noise intensity $D = 0.5As^{1/2}/mm^2$ and Lorentzian fit (thick line)

The Fourier transform of the autocorrelation function (10) is, by the Wiener-Khinchin theorem, the power spectrum of the current density. From (11) we obtain approximately a Lorentzian shaped power spectral density,

$$S(\omega) \propto \frac{\omega^2}{(\omega^2 - \omega_0^2)^2 + (\frac{2\omega}{t_e})^2}, \tag{12}$$

and $1/t_e$ is now simply the half-width of the spectral peak. Thus, more regular motion will be characterized by larger correlation times and smaller values of the spectral half-widths. From Fig. 4 we see that the Lorentzian approximation (12) can indeed be used to reproduce the main peak of the observed power spectrum of noise-induced oscillations in the superlattice.



(a) (b)

FIG. 5 (a) Main spectral peak of the power spectral density $S(2\pi f)$ vs. frequency f for increasing noise (from top to bottom: $D = 0.3, D = 0.5$ and $D = 1.0As^{1/2}/mm^2$). (b) Basic period T_0 *vs* noise intensity D.

To understand how the noise level in the supperlattice affects the essential characteristics of noise-induced oscillations we consider spectra for different values of noise intensities D (Fig. 5(a)). We see that an increase of the noise level broadens the spectral peak, leading to a decreased correlation time t_{cor} (Fig. 6). At the same time the position of the main spectral peak, corresponding to the basic frequency $\omega_0 = 2\pi/T_0$ of the oscillations, is almost unchanged. This is confirmed by Fig. 5(b), where the dependence of the basic period T_0 (the inverse of the frequency at which the spectral peak is centered) of the noisy oscillations versus the noise intensity is presented. This basic period is close to the period of self-oscillations above the Hopf bifurcation.

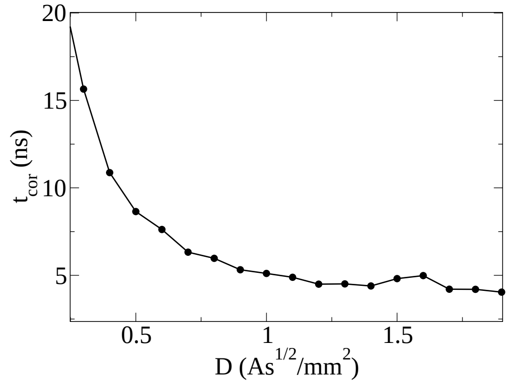


FIG. 6 Correlation time t_{cor} *vs* noise intensity D.

4. CONTROL OF NOISE-INDUCED DYNAMICS BY DELAYED FEEDBACK.

In contrast to the problem of controlling deterministic chaos, for which a number of methods have been proposed and successfully applied [Schuster, 1999], control of noise-induced motion is a significantly less studied concept. Previous works mainly concentrate on the control of stochastic oscillations in *low-dimensional* simple models [Balanov *et al.*, 2004; Christini & Collins, 1995; Janson *et al.*, 2004; Landa *et al.*, 1997; Masoller, 2002; Schöll *et al.*, 2005], or self-oscillations in the presence of noise [Goldobin *et al.*, 2003], while control of noise-induced dynamics in *spatially extended systems* seems still to be an open problem.

In this Section, we study the effect of a time delayed delayed feedback of the form

$$F(t) = K(s(t) - s(t - \tau)). \tag{13}$$

Time delayed feedback of this form is well studied in the field of chaos control [Schuster, 1999], and was proposed in [Janson *et al.*, 2004] for the improvement of the coherence of noise-induced oscillations in low-dimensional systems. Here $s(t)$ is an output signal of the system, τ is a delay time, and K is the feedback strength.

An easy way to implement control in the superlattice model is to choose the output signal to be the total current density, $s(t) = J(t)$ and simply add the control force to the external voltage U, i.e.

$$U = U_0 - K(J(t) - J(t - \tau)) \tag{14}$$

where U_0 is the time-independent external voltage bias. This control scheme is depicted in Fig. 7. Since both voltage and total current density are externally accessible global variables, such a control scheme is easy to implement experimentally.

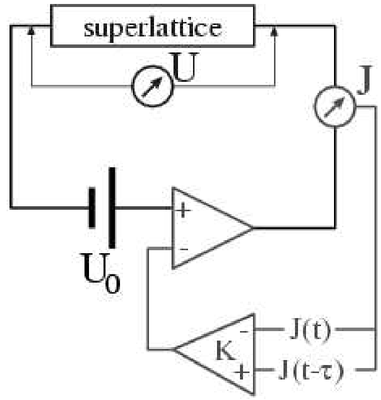
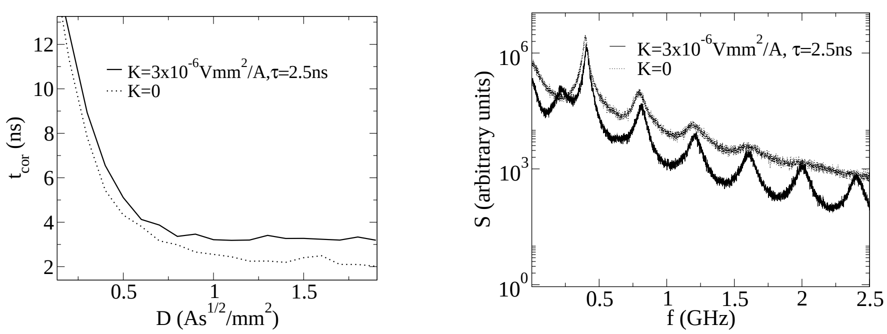


FIG. 7 Control circuit with time-delayed feedback loop

We now study the effect of applying the control force of the form (14) to the dynamical set of Eqs. (1)–(4). A natural choice for τ is the basic period of the Hopf oscillation (or integer multiples of it). In Fig. 8(a) we see that the application of the control force indeed improves the coherence of the current signal, in particular for large noise level D. From Fig. 8(b) we can conclude that the reason for this behavior is that the main peak in the power spectrum becomes narrower, though at the same time additional harmonic peaks occur.



(a) (b)

FIG. 8 (a) Correlation time t_{cor} *vs* noise intensity D: without delayed feedback (solid line), and with delayed feedback with $K = 3 \times 10^{-6} Vmm^2/A$ and $\tau = 2.5ns$ (dotted line). (b) Power spectral density of noise-induced oscillations with and without control.

From a practical point of view it is important to estimate the magnitude of the control force (13), which influences the system due to the delayed feedback. In Fig. 9 the expectation value of F^2 is plotted as a function of the time delay. While this quantity never vanishes, it demonstrates an oscillating behavior having pronounced minima at those values of τ which are multiples of the basic period of noise-induced oscillations without control.

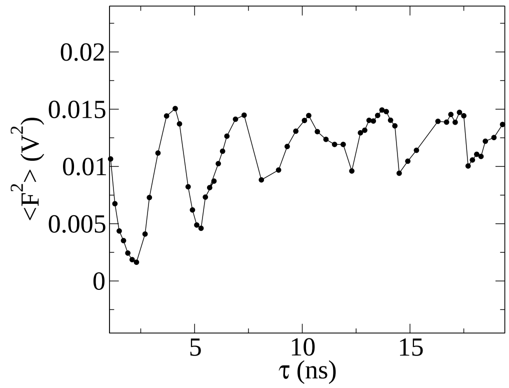


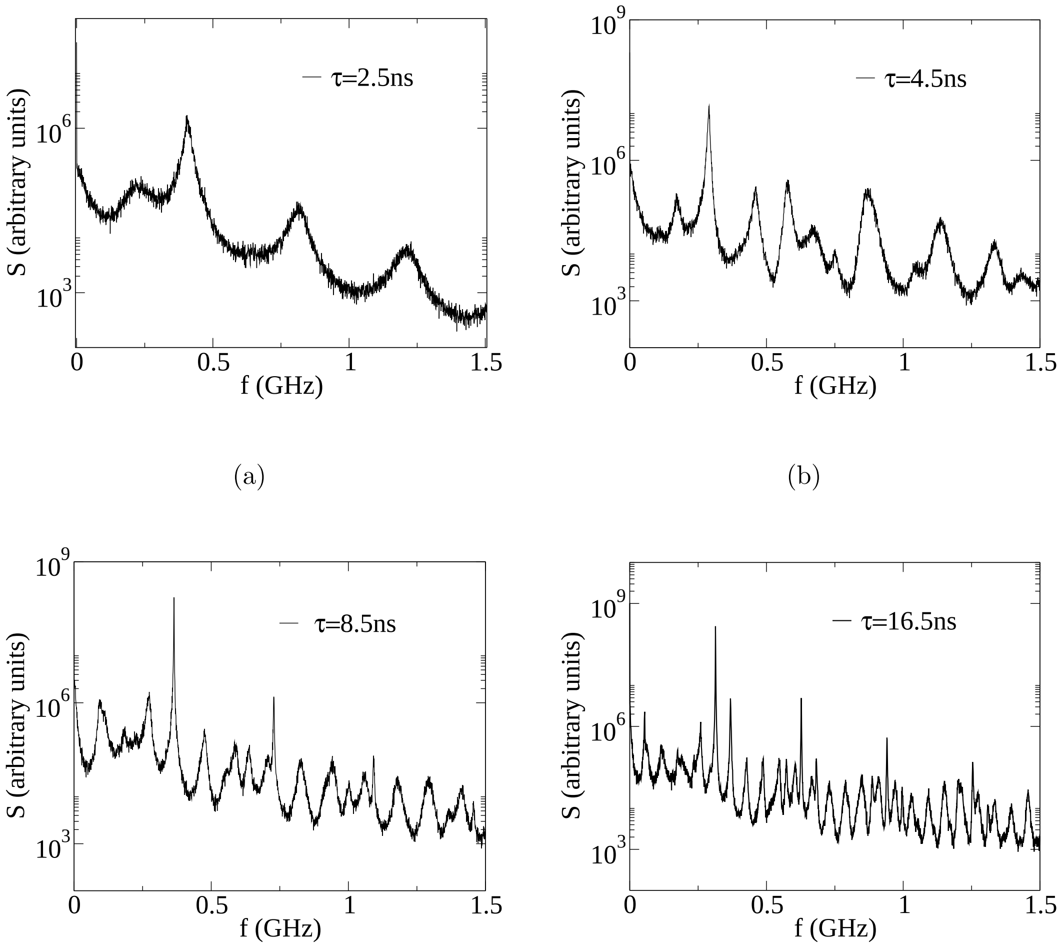
FIG. 9 Expectation value of the squared control force $\langle F^2 \rangle$ *vs* time delay τ for $K = 3 \times 10^{-6} Vmm^2/A$ and $D = 0.5$.

Next, we study the influence of varying τ upon the power spectra of the system. As is seen from Fig.10, with increasing τ additional peaks at the corresponding lower frequencies appear, while the main (most pronounced) peak moves towards lower frequencies. This is even more evident from Fig. 11, where the period of the resulting main peak as a function of τ is plotted. We see that for a large range, $T_0(\tau)$ has an almost piecewise linear, oscillatory character. Similar behavior has been found for the simple models without spatial degrees of freedom and explained analytically via a linear stability analysis [Balanov *et al.*, 2004; Janson *et al.*, 2004] or via the power spectrum [Schöll *et al.*, 2005].

Thus we note that while the position of the main peak of the spectrum is insensitive to the noise level in the case without control, it is indeed possible to shift its position by the proposed time delayed feedback scheme.

5. DISCUSSION AND CONCLUSION

We have studied noise-induced patterns in the semiconductor superlattice model in the regime where the only deterministic stationary solution (without noise) is a steady state. We have shown that in this case noise can induce quite coherent oscillations of the global current in the device, which could be associated with the spatio-temporal dynamics of depletion and accumulation fronts of the carrier density. The variation of the noise level can substantially change the regularity of noise-induced oscillations, whereas their basic time scale is almost insensitive to noise (cf. Fig. 6 and Fig. 5(b), respectively).



(a) (b)

(c) (d)

FIG. 10 (a) Power spectral density $S(f)$ of the controlled system for increasing values of time delay τ: (a) $\tau = 2.5ns$, (b) $\tau = 4.5ns$, (c) $\tau = 8.5ns$ and (d) $\tau = 16.5ns$. ($D = 0.5As^{1/2}/mm^2$, $K = 3 \times 10^{-6} Vmm^2/A$)

A practical application of such a behavior may be the construction of a sensor, which encodes the level of noise in only one essential parameter of the output signal, namely its coherence. In this case the noise is not necessarily only the internal noise which is caused by shot noise, but may have other external sources, such as the temperature of the environment, the presence of a magnetc field, etc. For such an application it is advantageous that the mean frequency itself does not depend significantly on the noise level.

A method of controlling stochastic oscillations, generated by a semiconductor superlattice, by means of time delayed feedback has been investigated. It was shown that control not

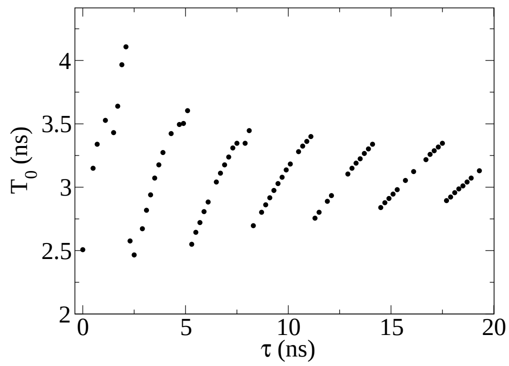


FIG. 11 Basic period as a function of the time delay τ. $K = 3 \times 10^{-6} Vmm^2/A$ and $D = 0.5$.

only enhances the regularity of motion but also allows us to manipulate the time scales of the system by varying the time delay τ. Control of noise-induced oscillations in this case is interesting from a practical point of view, in terms of electronic oscillators with tunable coherence properties and time scales, but also is of general theoretical interest, in terms of stochastic systems with memory.

6. ACKNOWLEDGEMENTS

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