1. Introduction

- Tunnel diodes show a region of negative differential conductivity (NDC) in current voltage characteristic, the current decreases with increasing voltage, current-voltage curves shapes like the letter "W" [1, 2].
- Here: two tunnel diodes connected in series (mean field coupling)
- Varying applied voltage \( \Rightarrow \) symmetry breaking transition (pitchfork bifurcation): synchronized, symmetric branch splits into a branch within synchronization manifold and two branches outside of synchronization manifold
- Hopf bifurcation occurs simultaneously with pitchfork bifurcation \( \Rightarrow \) each pitchfork branch surrounded by limit cycles [3]
- Introducing heterogeneity: Unfolding of Hopf-Pitchfork bifurcation

2. Tunnel diode model

Kirchhoff’s laws lead to the dynamical system:

\[
\begin{align*}
V_0 & = R I = \frac{1}{2} \sum_{i=1}^{2} V_i, \\
i & = C V_i + \frac{1}{3} V_i^3, \\
\dot{V}_i & = i(t) + f(V_i(t)),
\end{align*}
\]

with the cubic tunnel diode \( i-V \) curve: \( f(V_i(t)) \)

\[
f(V_i) = b_i \left( V_i - V_a - \frac{1}{3} (V_i - V_a)^3 \right).
\]

Scaling leads to FitzHugh-Nagumo like equations:

\[
\begin{align*}
V^{(i)} & \rightarrow x_i, \\
i^{(i)} & \rightarrow y_i, \\
V & \rightarrow z_i,
\end{align*}
\]

- \( d = \frac{R}{C} \) is ratio of load line \( R \) to maximum slope \( \rho \) of NDC region
- Bias voltage \( V_0 \) enters \( x^{(i)} = \frac{1}{2} (V_0 - NV_a - RD_a) \)
- \( \gamma \) and \( \epsilon \) are timescale parameters

\[
\begin{align*}
\epsilon & = 0.830, \\
\gamma & = 0.043, \\
\delta & = 0.675, \\
V_0 & = 4.547, \\
d & = 0.57.
\end{align*}
\]

3. Single tunnel diode

Bifurcation diagram and close up around subcritical Hopf bifurcation

- Parameter \( d \) changes shape of curve
- Timescale parameters \( \epsilon \) and \( \gamma \) may suppress Hopf bifurcation

4. Two tunnel diodes

Bifurcation diagram and close up around Hopf-pitchfork bifurcation

- Hopf-pitchfork bifurcation (HB, PF) at \( V_0 = 0.381V \)
- Pitchfork bifurcation of periodic orbits (PPO) at \( V_0 = 0.434V \)
- Parameter \( d \) changes subcritical to supercritical pitchfork bifurcation

Hopf-pitchfork bifurcation is a symmetry breaking transition

- Middle branch (\( \Delta V = 0 \)): isochronous dynamics
- Upper and lower branches: symmetry breaking, asymmetric fixed points, anti-phase orbits

Unfolding the Hopf-Pitchfork

Bifurcation diagram with difference in time scale parameters and parameter plane \( \Rightarrow \) One Hopf bifurcation along each branch

- Difference in bias voltage \( V_0^{(a)} \neq V_0^{(b)} \) \( \Rightarrow \) Imperfect pitchfork bifurcation

5. Comparison to measurements

Single tunnel diode

- Measured (top) and simulated (bottom) Voltage \( V \) for a single tunnel diode with ramped voltage source \( V_0 \)
- S: upwards ramp, red: downwards ramp

Two tunnel diodes

- Voltage \( V_0 \) ranges of identical and asymmetric fixed points
- Asymmetric fixed points evolve at symmetry breaking, covered by oscillations \( \Rightarrow \) Hopf-pitchfork bifurcation

6. Conclusion

- Tunnel diode dynamics can be described by extended FitzHugh-Nagumo model
- Two coupled diodes show complex bifurcation scenario: codimension three Hopf-pitchfork bifurcation and pitchfork bifurcation of periodic orbits
- Pitchfork bifurcation breaks symmetry and leads to asymmetric fixed points and anti-phase orbits
- Qualitatively good agreement between numerical and experimental results for one and two tunnel diodes

References


martin.heinrich@physik.tu-berlin.de