USING TIME-DELAYED FEEDBACK FOR CONTROL OF DYNAMICS IN COUPLED COHERENCE RESONANCE OSCILLATORS

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0. Introduction

We study the influence of delayed feedback on the cooperative noise-induced dynamics of two mutually coupled excitable systems, each forced by its own source of random fluctuations. As a model of interacting neurons we consider excitable FitzHugh-Nagumo systems. We show that by application of delayed feedback to one of these interacting systems one can deliberately change coherence of noise-induced oscillations and synchronization of the two systems.

1. Model

Two coupled noisy FitzHugh-Nagumo systems with time-delayed feedback control.

$$\epsilon_1 \dot{x}_1 = x_1 - \frac{x_1^3}{3} - y_1 + C(x_2 - x_1)$$
$$\dot{y}_1 = x_1 + a + \underbrace{D_1 \xi_1(t)}_{noise} + \underbrace{K[y_1(t-\tau) - y_1(t)]}_{control} \qquad (1)$$

$$\epsilon_2 \dot{x}_2 = x_2 - \frac{x_2^3}{3} - y_2 + C(x_1 - x_2)$$
$$\dot{y}_2 = x_2 + a + D_2 \xi_2(t) \qquad (2)$$

ξ_i : Independent Gaussian white noise sources

D_i : Noise intensities, define mean period of oscillations

τ : Time delay

K : Feedback strength

C : Coupling strength

$a = 1.05,\ \epsilon_1 = 0.005,\ \epsilon_2 = 0.1$

2. Properties without control, $K = 0$

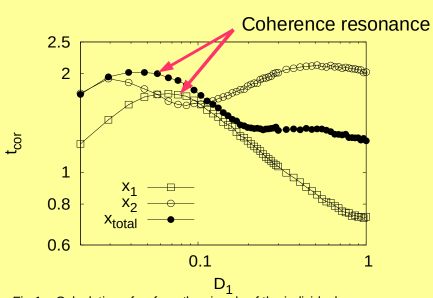
a) Regularity

The regularity of spiking is described by the correlation time

$t_{cor} = \frac{1}{\sigma^2}\int_0^\infty |\psi(s)|\, ds$, where

$\psi(s) = \langle [x(t-s) - \langle x \rangle][x(t) - \langle x \rangle] \rangle$ Autocorrelation function of signal $x(t)$

σ^2 Variance



$D_2 = 0.09$
$C = 0.07$

$x_{total} = x_1 + x_2$

Fig 1: Calculation of t_{cor} from the signals of the individual subsystems (x_1, x_2) and globally (x_{total}) vs. D_1

b) Calculation of ratio of mean interspike intervals (*ISI*) as a measure for stochastic synchronization.

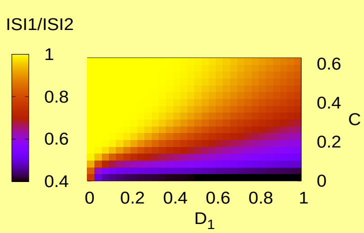
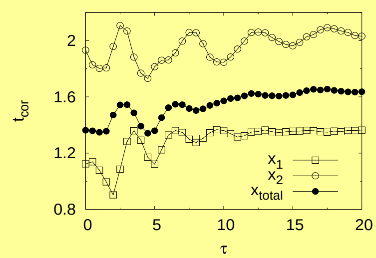


Fig 2: Ratio of mean interspike intervals vs. C and D_1

One can clearly see the synchronization region (yellow area).

3. Control of regularity

Application of feedback control is able to influence the regularity of oscillations either in individual subsystems or globally



$K = 0.2$
$C = 0.07$
$D_1 = 0.25$
$D_2 = 0.09$

Fig 3: Calculation of t_{cor} from the signals of the individual subsystems (x_1, x_2) and globally (x_{total}) vs. the time delay

The correlation times oscillate with increasing τ and saturates at large τ . The value of t_{cor} can be enhanced and decreased by choosing the appropriate control parameters τ and K.

4. Control of synchronization

The efficiency of the control depends on the choice of the parameters D_1 and C, i.e. on the position with respect to stochastic synchronization region (Fig. 2).

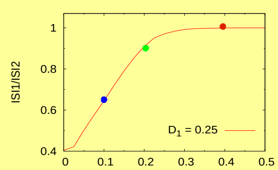


Fig 4a): Ratio of interspike intervals vs. the coupling strength C without control

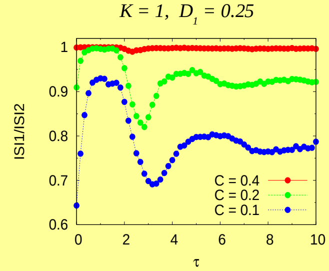
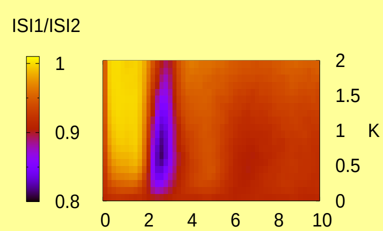


Fig 4b): Ratio of interspike intervals vs. τ for different values of the coupling strength C

$C = 0.4$ parameters well in synchronization region (SR), almost no effect

$C = 0.2$ parameters slightly outside of SR, induction and destruction of synchronization

$C = 0.1$ parameters well outside SR, tendency to synchronization



$D_1 = 0.25$
$D_2 = 0.09$
$C = 0.07$

Fig 5: Ratio of interspike intervals vs. the control strength K and the time delay τ

By choosing appropriate control parameters τ and K one can enhance (yellow areas) and destroy (blue/black areas) synchronization.

5. Conclusions

- In the uncontrolled systems synchronization is observed for certain parameters
- While applying delayed feedback control one can influence the correlation of oscillations by changing the time delay τ
- Synchronization of the two neurons can be enhanced or destroyed by choosing the appropriate control parameters