Growth kinetics and nonlinear transport for ensembles of quantum dots

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Strain field from elasticity theory

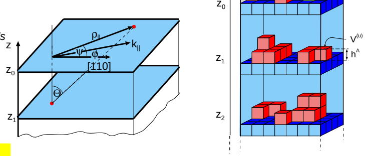
elastic energy of interacting inclusions

$$E_{el}(\mathbf{r}_\parallel; z=z_0) = V^{(u)} \int_A \int_{-\infty}^{z_0} d\mathbf{r}'_\parallel dz' W(\mathbf{r}_\parallel - \mathbf{r}'_\parallel; z_0 - z')$$

A. G. Khachaturyan: *Theory of Structural Transformations in Solids* (Wiley, New York, 1983).

$$W(\boldsymbol{r}_\parallel - \boldsymbol{r}'_\parallel; z_0 - z_1) = \int \frac{d^2\boldsymbol{k}_\parallel}{(2\pi)^2} \left\{ \sigma_{ij}^{(0)S} \left[\nabla_j \nabla'_m \tilde{G}_{il}(\boldsymbol{k}_\parallel; z, z') \right] \Big|_{\substack{z=z_0 \\ z'=z_0-z_1}} \sigma_{lm}^{(0)B} \right\} e^{i\boldsymbol{k}_\parallel(\boldsymbol{r}_\parallel - \boldsymbol{r}'_\parallel)}$$

numerical solution of Green's tensor $\tilde{G}_{ij}$ in k-space [POR77]



2D strain

analytical solution for interacting islands in one plane

$$E_{el}^{2D}(z=z_0) = \frac{h^S V^{(u)}}{c_{11}} \left(\frac{\Delta a}{a}\right)^2 \sigma^{(0)S} \sigma^{(0)B} \times \int\int d\rho_\parallel d\psi \left(\frac{1}{2\pi\rho_\parallel^3} [-A_0 + 15 B_0 \cos(4\psi)] \right)$$

3D strain

numerical solution for interaction between surface- and buried islands

$$E_{el}^{3D}(z > z_0) = \dots$$

additional energy barriers

$$E_{Str} = E_{el}^{2D} + E_{el}^{3D}$$

- h^S height of surface island
- $\Delta a / a$ lattice mismatch
- $\sigma^{(0)S,B}$ intrinsic stress
- c_{11} elastic constant
- $V^{(u)}$ volume of unit cell
- $\rho_\parallel = r_\parallel - r'_\parallel$

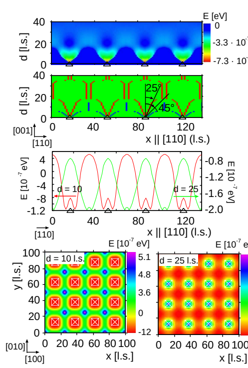
[POR77] K. Portz and A. A. Maradudin, Phys. Rev. B **16**, 3535 (1977)

similar method is used in:

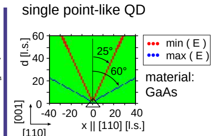
I. P. Ipatova, V. G. Malyshkin, and V. A. Shchukin, J. Appl. Phys. **74**, 7198 (1993).
V. A. Shchukin, D. Bimberg, V. G. Malyshkin, and N. N. Ledentsov, Phys. Rev. B **57**, 12262 (1998).

Strain field of buried structures

array of point defects



single point-like QD



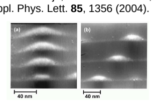
agreement with theory

V. Holy, G. Springholz, M. Pinczolits, and G. Bauer. Phys. Rev. Lett. **83**, 356 (1999).

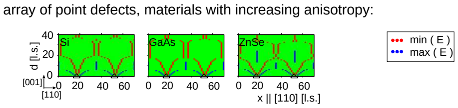
disagrees

agreement with experiments

X.-D. Wang, N. Liu, C. K. Shih, S. Govindaraju, and A. L. Holmes, Jr. Appl. Phys. Lett. **85**, 1356 (2004).

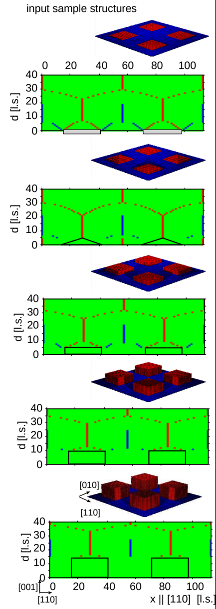


array of point defects, materials with increasing anisotropy:



arrays of QDs of different shapes:

input sample structures



R. Kunert and E. Schöll, Appl. Phys. Lett. (2006), in print. cond-mat/0607016

Kinetic Monte Carlo simulation

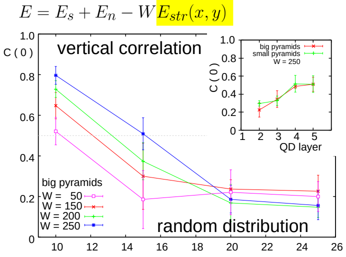
growth of QD stacks on prepatterned substrate

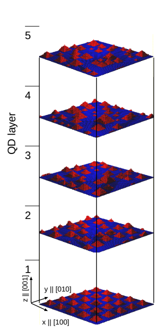
model of activated diffusion

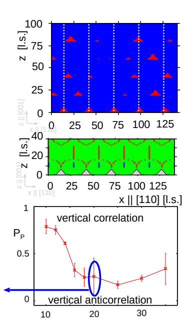
probability given by Arrhenius law

$$p = \nu_0 \exp\left(-\frac{E}{k_b T}\right)$$ where

$$E = E_s + E_n - W E_{str}(x,y)$$



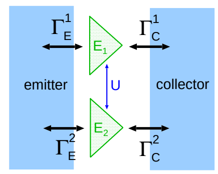




Tunneling through coupled QDs

- Master equation $\underline{\dot{P}} = \underline{\underline{\mathbf{M}}}\,\underline{P}$ with $\underline{P} = (P_{\{0,0\}}, P_{\{1,0\}}, P_{\{0,1\}}, P_{\{1,1\}})^T$ and

$$\underline{\underline{\mathbf{M}}} = \begin{pmatrix} -\Gamma_E^1 f_E^1 - \Gamma_E^2 f_E^2 & \Gamma_E^1(1-f_E^1) + \Gamma_C^1 & \Gamma_E^2(1-f_E^2) + \Gamma_C^2 & 0 \\ \Gamma_E^1 f_E^1 & -\Gamma_E^1(1-f_E^1) - \Gamma_C^1 - \Gamma_E^2 f_E^{2,U} & 0 & \Gamma_E^2(1-f_E^{2,U}) + \Gamma_C^2 \\ \Gamma_E^2 f_E^2 & 0 & -\Gamma_E^1 f_E^{1,U} - \Gamma_E^2(1-f_E^2) - \Gamma_C^2 & \Gamma_E^1(1-f_E^{1,U}) + \Gamma_C^1 \\ 0 & \Gamma_E^2 f_E^{2,U} & \Gamma_E^1 f_E^{1,U} & \Gamma_E^1 f_E^{1,U} + \Gamma_E^2 f_E^{2,U} - \Gamma \end{pmatrix}$$



with

$$\Gamma \equiv \Gamma_C^1 + \Gamma_C^2 + \Gamma_E^1 + \Gamma_E^2$$
$$f_E^i = (1 + \exp((E_i - e\eta V)/(k_B T)))^{-1}$$
$$f_E^{i,U} = (1 + \exp((E_i + U - e\eta V)/(k_B T)))^{-1}$$

Bias voltage $eV = \mu_E - \mu_C$

We consider high-bias regime: $eV \gg k_B T$ → $f_C^i = f_C^{i,U} = 0$

- Steady-state solution $\underline{\underline{\mathbf{M}}}\,\underline{\mathbf{P}}^0 = 0$

G. Kießlich, A. Wacker, and E. Schöll, Physica B 314, 459 (2002)

- Current Operators (Project the occupation probability to the state after an electron traversed the barrier.)

$$\underline{\underline{\mathbf{j}}}_C = e \begin{pmatrix} 0 & \Gamma_C^1 & \Gamma_C^2 & 0 \\ 0 & 0 & 0 & \Gamma_C^2 \\ 0 & 0 & 0 & \Gamma_C^1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \underline{\underline{\mathbf{j}}}_E = e \begin{pmatrix} 0 & -\Gamma_E^1(1-f_E^1) & -\Gamma_E^2(1-f_E^2) & 0 \\ \Gamma_E^2 f_E^2 & 0 & 0 & -\Gamma_E^1(1-f_E^{1,U}) \\ \Gamma_E^1 f_E^1 & 0 & 0 & -\Gamma_E^2(1-f_E^{2,U}) \\ 0 & \Gamma_E^1 f_E^{1,U} & \Gamma_E^2 f_E^{2,U} & 0 \end{pmatrix}$$

- Stationary Current $\langle I \rangle = \sum_\nu [\underline{\underline{\mathbf{j}}}_C \underline{\mathbf{P}}^0]_\nu = \sum_\nu [\underline{\underline{\mathbf{j}}}_E \underline{\mathbf{P}}^0]_\nu$

- Time propagator $\underline{\underline{\mathbf{T}}}(t) \equiv \exp(\underline{\underline{\mathbf{M}}} t)$ with $\underline{\mathbf{P}}(t) = \underline{\underline{\mathbf{T}}}(t)\,\underline{\mathbf{P}}(0)$

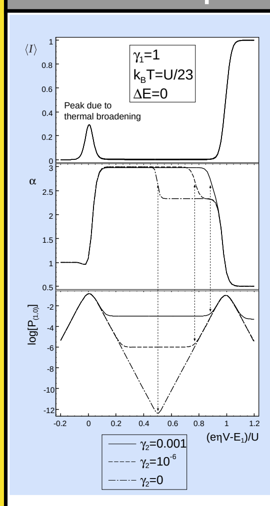
- Correlation Function

$$\langle I_a(t) I_b(0) \rangle = \theta(t) \sum_\nu [\underline{\underline{\mathbf{j}}}_a \underline{\underline{\mathbf{T}}}(t) \underline{\underline{\mathbf{j}}}_b \underline{\mathbf{P}}^0]_\nu + \theta(-t) \sum_\nu [\underline{\underline{\mathbf{j}}}_b \underline{\underline{\mathbf{T}}}(-t) \underline{\underline{\mathbf{j}}}_a \underline{\mathbf{P}}^0]_\nu + e\delta_{ab}\delta(t) \sum_\nu \left| [\underline{\underline{\mathbf{j}}}_{a/b} \underline{\mathbf{P}}^0]_\nu \right|$$

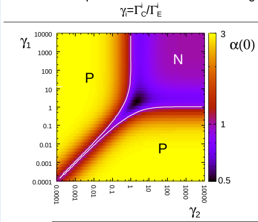
- Spectral power density $S_{ab}(\omega) = 2 \int_{-\infty}^{\infty} dt e^{i\omega t} \left(\langle I_a(t) I_b(0) \rangle - \langle I \rangle^2 \right)$

Fano factor $\alpha \equiv \dfrac{S_{ab}(0)}{2e\langle I \rangle}$

Super-Poissonian noise



Fano factor α dependence on ratios of tunneling rates $\gamma_i = \Gamma_C^i / \Gamma_E^i$



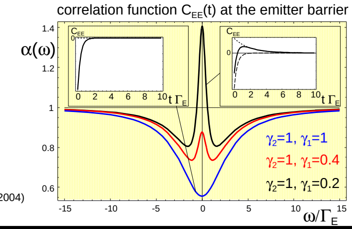
$\Gamma_E^1 = \Gamma_E^2 = \Gamma_E$ (equal emitter couplings)

N...negative correlations (Pauli blocking)

P...positive correlations (bunching due to Coulomb blocking)

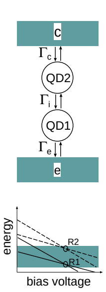
$$\alpha(0) = 1 + \frac{2\left[\gamma_1^2 + \gamma_2^2 - \gamma_1\gamma_2(2 + \gamma_1 + \gamma_2)\right]}{(\gamma_1 + \gamma_2 + \gamma_1\gamma_2)^2}$$

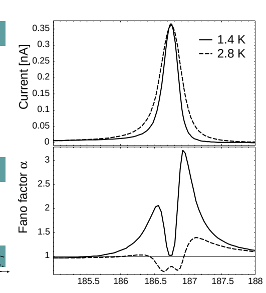
Frequency-dependent Fano factor and correlation function $C_{EE}(t)$ at the emitter barrier

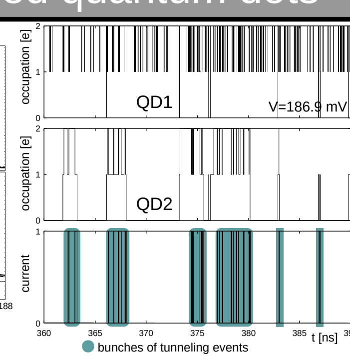


G. Kießlich et al, PRB 68, 12530 (2003), SST 19, S37 (2004)

Vertically coupled quantum dots







G. Kießlich et al, Proc. HCIS-14 Chicago (2005)

Collaborations: A4, A5, A7, B1, B14