Comment on "Ferromagnetic Microswimmers"

In their Letter entitled "Ferromagnetic Microswimmers" [1], Ogrin *et al.* propose a model for a microswimmer that consists of two magnetic beads connected by a spring. The idea is to generate directed motion by applying a time-dependent magnetic field that varies the magnetic moments in time and thereby causes a nonreciprocal time dependence of torques and forces acting on the dumbbell. Although it is generally possible to construct a swimmer on these premises, it can only work if there is a hydrodynamic interaction between the two beads. In this Comment we will show that no net motion is possible if the hydrodynamics is described with a separate Stokes friction term for each bead, as stated in the Letter ($\mathbf{F}_D = -6\pi\eta R v$). This contradicts the drift motion of the swimmer seen in the Letter.

The magnetic dipole forces, as well as those exerted by the spring, always act in pairs. The force on the first particle is exactly the opposite from the force on the second particle, $\mathbf{F}_1 = -\mathbf{F}_2$. Here, $\mathbf{F}_j$ represents the sum of the magnetic force $\mathbf{F}_{M,j}$, the radial component of the spring force $\mathbf{F}_{R,j}$ and its lateral component (constraint force) $\mathbf{F}_{C,j}$. For the magnetic force, $\mathbf{F}_{M,1} = -\mathbf{F}_{M,2}$ is a property of the dipole-dipole interaction. For the spring forces (longitudinal as well as lateral component), this follows from the fact that the sum of all forces acting on the spring has to vanish. The magnetic torques acting on the particles determine the lateral components of the spring force but no detailed knowledge about them is necessary for our argument. Therefore, the sum of all forces (excluding friction) on both particles is zero, $\mathbf{F}_1 + \mathbf{F}_2 = 0$. Because the motion is overdamped and inertia can be neglected, the viscous drag on each particle is the opposite of the sum of all other forces, $\mathbf{F}_{D,j} = -\mathbf{F}_j = -(\mathbf{F}_{M,j} + \mathbf{F}_{R,j} + \mathbf{F}_{C,j})$.

We can now define a *center of reaction* [2], which plays a similar role in low Reynolds number hydrodynamics as the center of mass in Newtonian mechanics:

$$\mathbf{x}_C = \frac{\sum_{j=1}^{2} \gamma_j \mathbf{x}_j}{\sum_{j=1}^{2} \gamma_j} \tag{1}$$

where $\gamma_j = 6\pi\eta R_j$. It follows $\dot{\mathbf{x}}_C = (\sum_{j=1}^{2} -\mathbf{F}_{D,j})/(\sum_{j=1}^{2} \gamma_j) = 0$. The center of reaction will therefore never change its position in this model.

A very convenient method to treat systems with internal constraints is Lagrangian mechanics [3]. Restricting the swimmer to two dimensions, the generalized coordinates of the dumbbell are the two position variables of the center of reaction $q_{1/2} = x_{C1/2}$, the distance between the beads $q_3 = r_{12}$, the angle the dumbbell encloses with, e.g., the x direction, $q_4 = \theta$, and the angle between $\mathbf{m}_1$ and the dumbbell axis $q_5 = \alpha$. At low Reynolds numbers, all inertial terms can be neglected and the equations of motion for the five generalized coordinates q_β read [3]

$$-\frac{\partial L}{\partial q_\beta} = \frac{\partial U}{\partial q_\beta} = Q_\beta = \sum_{j=1}^{2} \mathbf{F}_{D,j} \cdot \frac{\partial \mathbf{x}_j}{\partial q_\beta}. \tag{2}$$

Here the potential energy $U = U_R + U_M + U_F + U_A$ consists of the elastic energy of the spring, $U_R = k_s(r_{12} - r_0)^2/2$, the interaction energy of the magnetic dipoles $U_M = (\mu_0/4\pi)[\mathbf{m}_1 \cdot \mathbf{m}_2/r_{12}^3 - 3(\mathbf{m}_1 \cdot \mathbf{r}_{12})(\mathbf{m}_2 \cdot \mathbf{r}_{12})/r_{12}^5]$, where $\mathbf{r}_{12} = (r_{12}\cos\theta, r_{12}\sin\theta)$, the interaction of the magnetic dipole $\mathbf{m}_1$ with the field, $U_F = -\mu_0 \mathbf{m}_1 \cdot \mathbf{H}$ (Note that in the concrete implementation of Ref. [1], $\mathbf{m}_2$ follows the external field $\mathbf{H}(t)$ instantaneously and therefore does not appear in U_F) and the anisotropy term $U_A = K\sin^2\alpha$. Q_β are the generalized friction forces and the generalized coordinates are related to the position vectors of the beads via $\mathbf{x}_{1/2} = \mathbf{x}_C \mp \gamma_{2/1}\mathbf{r}_{12}/(\gamma_1 + \gamma_2)$. Since U does not depend on $\mathbf{x}_C$ and with $\mathbf{F}_{D,j} = -\gamma_j \dot{\mathbf{x}}_j$, Eq. (2) gives $Q_{1/2} = -(\gamma_1 + \gamma_2)\dot{x}_{C1/2} = 0$; so the center of reaction does not move as argued before. The equation that determines the direction of $\mathbf{m}_1$ is $Q_5 = -\mu_0(\mathbf{m}_1 \times \mathbf{H})_z + K\sin(2\alpha) = 0$ (as in [1] we neglect $\partial U_M/\partial\alpha$). For the remaining equations of motion one derives:

$$\dot{r}_{12} = (\gamma_1^{-1} + \gamma_2^{-1})\Big[k_s(r_0 - r_{12}) + \frac{\mu_0}{4\pi}\Big(3\frac{\mathbf{m}_1 \cdot \mathbf{m}_2}{r_{12}^4} - 9\frac{(\mathbf{m}_1 \cdot \mathbf{r}_{12})(\mathbf{m}_2 \cdot \mathbf{r}_{12})}{r_{12}^6}\Big)\Big] \tag{3}$$

$$\dot{\theta} = (\gamma_1^{-1} + \gamma_2^{-1})\mu_0(\mathbf{m}_1 \times \mathbf{H})_z/r_{12}^2. \tag{4}$$

We numerically solved these equations using parameters from Ref. [1]. The dumbbell rotates about the center of reaction without a noticeable variation of the distance r_{12}.

If hydrodynamic interactions are included, the friction force on bead j is $\mathbf{F}_{D,j} = -\sum_k \boldsymbol{\gamma}_{jk}\dot{\mathbf{x}}_k$ and one arrives at generalized friction forces $(Q_1, Q_2) = -\sum_{jk}\boldsymbol{\gamma}_{jk}(r_{12}, \theta)\dot{\mathbf{x}}_k = 0$. Because of the hydrodynamic interactions described by the friction matrices $\boldsymbol{\gamma}_{ij}$, $\mathbf{x}_C$ now couples to r_{12} and θ and therefore is able to display a drift in time.

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