7.5. Gaußscher Satz

• **Satz:** Quellen von $\underline{a}$ in V

$$\int_V \operatorname{div} \underline{a}\, dV = \int_{\partial V} \underline{a} \cdot d\underline{f} \qquad (7.34)$$

... Fluß durch Oberfläche ∂V

wichtig: (1) $\operatorname{div} \underline{a}$ definiert in ganz V

(2) $d\underline{f}$ zeigt aus V heraus

• Beweis:

1. Nähere V durch "viele" Quader ΔV_i 

"keine Zeichnung"

2. $$\int_V \operatorname{div} \underline{a}\, dV = \sum_{i} \operatorname{div} \underline{a}(\underline{r}_i)\, \Delta V_i$$
(über alle Quader)

3. $$\operatorname{div} \underline{a}(\underline{r}_i)\, \Delta V_i = \text{"}\int_{\partial(\Delta V_i)} \underline{a} \cdot d\underline{f}^{(i)}\text{"} \qquad \text{[vgl. Kap 6.5, Gl. (6.43)]}$$

4. benachbarte Vol. elemente:

$d\underline{f}^{(j)}$ $\quad d\underline{f}^{(i)} = -d\underline{f}^{(j)} \longrightarrow \underline{a} \cdot \underbrace{d\underline{f}^{(i)}}_{-d\underline{f}^{(j)}} + \underline{a} \cdot d\underline{f}^{(j)} = 0$

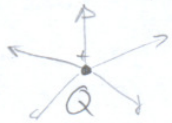
also: in $\sum_i \operatorname{div} \underline{a}(\underline{r}_i)\, \Delta V_i = \sum_i \text{"}\oint_{\partial(\Delta V_i)} \underline{a} \cdot d\underline{f}^{(i)}\text{"}$

nur "freiliegende" Oberflächen der ΔV_i tragen bei

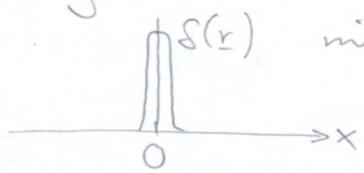
$\longrightarrow$ Oberfläche ∂V von V

$$\longrightarrow \sum_i \text{"}\oint_{\partial(\Delta V_i)} \underline{a} \cdot d\underline{f}^{(i)}\text{"} = \int_{\partial V} \underline{a} \cdot d\underline{f} \qquad \text{qed}$$

• Anwendung: E-Feld einer Pkt. ladung Q



Maxwell: $\operatorname{div} \underline{E} \sim \underbrace{Q\,\delta(\underline{r})}_{\text{Ladungsdichte}}$ (7.35)



$\delta(\underline{r})$ mit $\int \delta(\underline{r})\, dV = 1$

(i) $\boxed{\underline{E} = E(r)\, \underline{e}_r}$! (7.36)

(ii) $\int_{V_K} \operatorname{div} \underline{E}\, dV \underset{(7.35)}{\sim} \int_{V_K} Q\,\delta(\underline{r}) = Q$

Kugel um $\underline{r} = 0$

(iii) $\int_{\partial V_K} \underline{E} \cdot d\underline{f} \underset{(7.36)}{=} \int_{\partial V_K} E(\underline{r})\, \underline{e}_r \cdot \underline{e}_r\, df = \int_{\partial V_K} E(r)\, df$

$\overset{r = \text{const!}}{=} E(r) \int_{\partial V_K} df = E(r)\, 4\pi r^2$

Gauß: (ii) = (iii) $\longrightarrow$ $\boxed{E(r) \sim \frac{Q}{r^2}}$! (7.37)