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www.itp.tu-berlin.de/stat-i-ss07.html

3. Übung/Projekt zur Stat. Phys. I, computational methods

Path Integrals. Numerical Renormalisation Group (NRG)

Exercise

- Calculate the propagator of the harmonic oscillator in one dimension by directly 'doing the path integral'. You may get some help from L. S. Schulman's book.
- Revise Wilsons arguments leading to the logarithmic discretisation of the conduction band in his calculation of the Kondo problem within the NRG.
- Check the recursion relation given in the lecture notes,

$$H_{N+1} = \sqrt{\Lambda} H_N + \sum_\sigma \xi_N \left(f^\dagger_{N\sigma} f_{N+1\sigma} + f^\dagger_{N+1\sigma} f_{N\sigma} \right).$$

- Check the first part of the Lanczos method (cf. lecture notes),

$$\begin{aligned} \langle 0|H|0\rangle &= 0 \\ \lambda_0^2 &\equiv \langle 0|H^2|0\rangle = \frac{1}{4}\frac{(1+\Lambda^{-1})^2(1-\Lambda^{-1})}{1-\Lambda^{-3}}. \end{aligned}$$

Project III

Write a Numerical Renormalisation Code for the calculation of the low-energy spectrum of the Kondo-Hamiltonian. Extent your code to calculate thermodynamic quantities (spin-susceptibility, specific heat).

Project IV

Re-derive Wilson's expressions for the coefficients λ_n in the Wilson chain (cf. lecture notes)

$$\begin{aligned} \mathcal{H}_K &= 2J\mathbf{S}\sum_{\mu\nu} f^\dagger_{0\mu}\vec{\sigma}_{\mu\nu} f_{0\nu} + \sum_{n=0\sigma}^{\infty} \lambda_n \left(f^\dagger_{n\sigma} f_{n+1\sigma} + f^\dagger_{n+1\sigma} f_{n\sigma} \right) \\ \lambda_n &= \frac{\Lambda^{-n/2}}{2}\left[1-\Lambda^{-(n+1)}\right]\left[1-\Lambda^{-2n-1}\right]^{-1/2}\left[1-\Lambda^{-(2n+3)}\right]^{-1/2}\left[1+\Lambda^{-1}\right], \end{aligned}$$

possibly by writing a MATHEMATICA code.

NOTE:
Exercises should be done by all participant. Projects are for those who wish to get a 'Schein', or for those who wish to apply the material in the Lecture Notes to some interesting problems. These projects are related to some recent research in our group.

Bitte Rückseite beachten!⟶

- **Internetseite der Veranstaltung:** http://www.itp.tu-berlin.de/stat-i-ss07.html
- **Vorlesung:** Mittwoch 12:15 - 14:00 Uhr im PN 201 und Donnerstag 14:15 - 16:00 Uhr im PN 731
- **Tutorium:** Dienstags 8:30 - 10:00 Uhr im PN 731
- **Scheinkriterien:** Erfolgreiche Teilnahme an den Übungen und erfolgreiche Durchführung eines Projektes
- **Sprechstunden:**
  - Prof. Dr. Tobias Brandes: Montags, 13:00 - 14:00 Uhr
  - Philipp Zedler: Mittwoch, 11:00 - 12:00 Uhr
- **Literatur:** s. Vorlesung.