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Technische Universität Berlin
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Dr. Clive Emary http://www.itp.tu-berlin.de/menue/lehre/lv/ss09/wpfv/statphys_i/

4. Übungsblatt zur Statistiche Physik I

Boltzmann Equation

Abgabe: Mittwoch, 20th May, bis 16:00 Uhr, Raum E-W 705

Exercise 12 (3 points): *Impurity scattering*
Consider a gas of non-interacting particles colliding elastically with a set of fixed scatterers of density n_0. With no external forces and the collision term written in the following manner (see Lecture),

$$\left.\frac{\partial f}{\partial t}\right|_{\text{coll.}} = \int d^3p_2 d\Omega \left|\frac{d\sigma}{d\Omega}\right| |\mathbf{v}-\mathbf{v}_2| \left[f(\mathbf{q},\mathbf{p}_3,t)f(\mathbf{q},\mathbf{p}_4,t) - f(\mathbf{q},\mathbf{p},t)f(\mathbf{q},\mathbf{p}_2,t)\right],$$

show that, for this special case, the Boltzmann equation reduces to

$$\left\{\frac{\partial}{\partial t} + \frac{1}{m}\mathbf{p}\cdot\boldsymbol{\nabla}_q\right\} f(\mathbf{q},\mathbf{p},t) = \frac{n_0|\mathbf{p}|}{m}\int d\Omega \frac{d\sigma}{d\Omega}\left[f(\mathbf{q},\mathbf{p_3},t) - f(\mathbf{q},\mathbf{p},t)\right], \tag{1}$$

with $f(\mathbf{q},\mathbf{p},t)$ the distribution function of the gas particles.

Exercise 13 (3 points): *Lorentz gas cross section*
Consider a two-dimensional gas with impurity scatterers modelled as hard circles of radius a. Show that for scattering through an angle θ, the impact parameter b, differential cross-section $d\sigma$, and total cross-section σ_{tot} are given by

$$b = a\cos\frac{\theta}{2}; \quad d\sigma = \frac{a}{2}\sin\frac{\theta}{2}d\theta; \quad \text{and } \sigma_{\text{tot}} = 2a.$$

Exercise 14 (4 points): *Diffusion Equation*
Defining the density $n(\mathbf{q},t) \equiv \int d^3p f(\mathbf{q},\mathbf{p},t)$ and current density $\mathbf{j}(\mathbf{q},t) \equiv \int d^3p \frac{\mathbf{p}}{m} f(\mathbf{q},\mathbf{p},t)$, show that Eq. (1) implies

$$\frac{\partial}{\partial t} n(\mathbf{q},t) + \boldsymbol{\nabla}_q \cdot \mathbf{j}(\mathbf{q},t) = 0.$$

With Fick's law $\mathbf{j}(\mathbf{q},t) = -D\cdot\boldsymbol{\nabla}_q n(\mathbf{q},t)$ where D is the diffusion coefficient, we obtain the diffusion equation

$$\frac{\partial}{\partial t} n(\mathbf{q},t) = D\nabla_q^2 n(\mathbf{q},t).$$

Show that the dispersion relation for these diffusive modes is

$$\omega = -iDk^2,$$

and interpret the result. **Bitte Rückseite beachten!**⟶

- **Internetseite der Veranstaltung:** http://www.itp.tu-berlin.de/menue/lehre/lv/ss09/wpfv/statphys_i/
- **Vorlesung:** Montags & Donnerstags, 14:15 bis 15:45, E-W 202
- **Literatur:**
  - D. A. McQuarrie, Statistical Mechanics
  - L. E. Reichl, A Modern Course in Statistical Mechanics
  - F. Schwabl, Statistische Mechanik
  - M. Kardar, Statistical Physics of Particles & Statistical Physics of Fields
  - M. Plischke and B. Bergersen, Equilibrium Statistical Physics
  - H. B. Callen, Thermodynamics and an Introduction to Thermostatistics
- **Übung:** Donnerstags, 10:15 bis 11:45, E-W 733
- **Scheinkriterien:** 50% der Punkte aus den Übungszetteln (Zweierabgabe), aktive Teilnahme an den Tutorien
- **Sprechstunden:**

  Prof. Dr. H. Stark: Fr. 11:30 - 12:30,E-W 709
  Dr. C. Emary: Di, 16:00 - 17:00 Uhr, E-W 705