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5. Übungsblatt zur Statistiche Physik I

Boltzmann's H-theorem and Irreversibility

Abgabe: Mittwoch, 3rd June, bis 16:00 Uhr, Raum E-W 705

Exercise 15 (3 points): *Maxwell-Boltzmann distribution*
Let

$$H = \int d^3p f(\mathbf{p},t) \ln f(\mathbf{p},t)$$

where $f(\mathbf{p},t)$ is arbitrary except for the conditions

$$\int d^3p f(\mathbf{p},t) = n \quad \text{and} \quad \int d^3p \frac{p^2}{2m} f(\mathbf{p},t) = \bar{\epsilon} = \frac{3}{2} n k_\mathrm{B} T$$

Show that H is minimum when f is the Maxwell-Boltzmann distribution.

Exercise 16 (7 points): *Ehrenfest dog-flea model*
Suppose we have two dogs and $2R$ fleas, numbered from 1 to $2R$. Initially let all $2R$ fleas be on one dog. Then draw a number from 1 to $2R$ and move the flea with that number onto the other dog. Let this process be continued, and let $N_A + N_B = 2R$ and $N_A - N_B = 2k$, where N_A and N_B are the number of fleas on each dog. If we draw and transfer a flea every τ seconds, then $t = s\tau$, with s the number of draws.

- Show that, in one draw, the probability for system to go from state k to $k \pm 1$ is

$$W_{k\pm1,k} = \frac{R \mp k}{2R}.$$

- Show that the probability $P(k,s)$ to find the system in state k after s draws can be described by the Markovian chain

$$P(k,s) = \frac{R+k+1}{2R} P(k+1, s-1) + \frac{R-k+1}{2R} P(k-1, s-1). \tag{1}$$

- Use Eq. (1) to show that the average value of k as a function of s is given by

$$\overline{k}(s) \equiv \sum_k k P(k,s) = \left(1 - \frac{1}{R}\right)^s \overline{k}(0),$$

  where $\overline{k}(0)$ is the initial value. Hence show that, in the large R limit deviations from $\overline{k} = 0$ are exponentially damped:

$$\overline{k}(t) \sim \overline{k}(0) e^{-\gamma t} \quad \text{with} \quad \gamma = (R\tau)^{-1}. \tag{2}$$

- Run a computer simulation of the model. Plot $N_A(s)$ and $N_B(s)$ for a typical run with initial conditions $N_A(0) = 1$ and $N_B(0) = 0$. Compare with the values of $\overline{N_A}(s)$ and $\overline{N_B}(s)$ obtained from Eq. (2) for $R = 5, 20, 100$.

- Using your simulation, determine the average time taken for the system to return to end up in state $k = R$ given that its initial state is $k = 0$. Plot this average time as a function of R. [Hint: start with small R and work up]. What does this calculation tell us about the 'reversibility paradox' (also known as Zermelo's or Loschmidt's paradox)?

Bitte Rückseite beachten!⟶

- **Internetseite der Veranstaltung:** http://www.itp.tu-berlin.de/menue/lehre/lv/ss09/wpfv/statphys_i/
- **Vorlesung:** Montags & Donnerstags, 14:15 bis 15:45, E-W 202
- **Literatur:**
  - D. A. McQuarrie, Statistical Mechanics
  - L. E. Reichl, A Modern Course in Statistical Mechanics
  - F. Schwabl, Statistische Mechanik
  - M. Kardar, Statistical Physics of Particles & Statistical Physics of Fields
  - M. Plischke and B. Bergersen, Equilibrium Statistical Physics
  - H. B. Callen, Thermodynamics and an Introduction to Thermostatistics
- **Übung:** Donnerstags, 10:15 bis 11:45, E-W 733
- **Scheinkriterien:** 50% der Punkte aus den Übungszetteln (Zweierabgabe), aktive Teilnahme an den Tutorien
- **Sprechstunden:**

  Prof. Dr. H. Stark: Fr. 11:30 - 12:30,E-W 709
  Dr. C. Emary: Di, 16:00 - 17:00 Uhr, E-W 705