Technische Universität Berlin
Institut für Theoretische Physik
Prof. Dr. Holger Stark,
Dr. Clive Emary http://www.itp.tu-berlin.de/menue/lehre/lv/ss09/wpfv/statphys_i/

8. Übungsblatt zur Statistiche Physik I

Virial Coefficients

Abgabe: Mittwoch, 24th June, bis 16:00 Uhr, Raum E-W 705

Exercise 23 (5 points): *Virial coefficients for a gas of hard spheres*
The second and third virial coefficients, $B_2(T)$ and $B_3(T)$ are defined as

$$\begin{aligned} B_2(T) &= -\frac{1}{2V}\int d\mathbf{r}_1 \int d\mathbf{r}_2 f(\mathbf{r}_{21}) \\ B_3(T) &= -\frac{1}{3V}\int d\mathbf{r}_1 \int d\mathbf{r}_2 \int d\mathbf{r}_3 f(\mathbf{r}_{21}) f(\mathbf{r}_{31}) f(\mathbf{r}_{32}) \end{aligned} \tag{1}$$

where $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$ and $f(\mathbf{r}) = (e^{-\beta V(\mathbf{r})} - 1)$. Calculate these coefficients for interacting hard spheres with $V(\mathbf{r}) = \infty$ for $|\mathbf{r}| < R$ and $V(\mathbf{r}) = 0$ for $|\mathbf{r}| > R$. [Hints: For B_2, change integration variables to $\mathbf{q} = \mathbf{r}_{21}$ and $\mathbf{Q} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)$. For B_3, change integration variables to $\mathbf{r}_1$, $\mathbf{q} = \mathbf{r}_{31} - \mathbf{r}_{21}$ and $\mathbf{Q} = \frac{1}{2}(\mathbf{r}_{31} + \mathbf{r}_{21})$; the integration over $\mathbf{Q}$ can be performed geometrically.]

Exercise 24 (1 points): *Second virial coefficient for an ideal Fermi gas*
The (quantum-mechanical) grand-canonical partition sum for a Fermi gas with eigenmode energies ϵ_k reads

$$\mathcal{Z}_G = \prod_{k=1}^{\infty} (1 + e^{\beta(\mu - \epsilon_k)}). \tag{2}$$

For free particles of mass m the eigenenergies are $\epsilon(\mathbf{k}) = \hbar^2 k^2/(2m)$.

- Show that in the continuum limit with box normalisation (volume V), Eq. (2) gives

$$\begin{aligned} \rho &= \frac{N}{V} = C\int_0^{\infty} d\epsilon \frac{\epsilon^{1/2}\lambda e^{-\beta\epsilon}}{1+\lambda e^{-\beta\epsilon}} \\ PV &= C k_B T V \int_0^{\infty} d\epsilon \epsilon^{1/2} \ln(1 + \lambda e^{-\beta\epsilon}) \end{aligned} \tag{3}$$

  with $\lambda = e^{\beta\mu}$ and $C = (2m/\hbar^2)^{3/2}/(2\pi^2)$ for spin-half particles.

- Assuming that λ is small, show that these quantities can be written in series form

$$\rho = \frac{1}{\Lambda^3}\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\lambda^n}{n^{3/2}} \tag{4}$$

$$\frac{P}{k_B T} = \frac{1}{\Lambda^3}\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\lambda^n}{n^{5/2}} \tag{5}$$

  and hence determine Λ.

- Partially revert the power series of Eq. (4) to find an expression for λ as a series in the density: $\lambda = a_0 + a_1\rho + a_2\rho^2 + \ldots$ (up to second order will suffice).

- Use these results to find the second virial coefficient in the expansion

$$\frac{p}{k_B T} = \rho + B_2(T)\rho^2 + \ldots \tag{6}$$

  for the ideal Fermi gas. What is the sign of $B_2(T)$? What is the physical interpretation?

Bitte Rückseite beachten!⟶

- **Internetseite der Veranstaltung:** http://www.itp.tu-berlin.de/menue/lehre/lv/ss09/wpfv/statphys_i/
- **Vorlesung:** Montags & Donnerstags, 14:15 bis 15:45, E-W 202
- **Literatur:**
  - D. A. McQuarrie, Statistical Mechanics
  - L. E. Reichl, A Modern Course in Statistical Mechanics
  - F. Schwabl, Statistische Mechanik
  - M. Kardar, Statistical Physics of Particles & Statistical Physics of Fields
  - M. Plischke and B. Bergersen, Equilibrium Statistical Physics
  - H. B. Callen, Thermodynamics and an Introduction to Thermostatistics
- **Übung:** Donnerstags, 10:15 bis 11:45, E-W 733
- **Scheinkriterien:** 50% der Punkte aus den Übungszetteln (Zweierabgabe), aktive Teilnahme an den Tutorien
- **Sprechstunden:**

  Prof. Dr. H. Stark: Fr. 11:30 - 12:30,E-W 709
  Dr. C. Emary: Di, 16:00 - 17:00 Uhr, E-W 705