

Kapitel 4

Zufallswege, Reibung und Diffusion

Das Diffusionsgesetz ist universell:

$\hat{=}$ Modell unabhängig

Bsp: Verteilung von Zufallsschritten

1D-Modell: Schrittlängenverteilung: kL ,
 $P_k = \text{const. für } k \in [k_a, k_e]$
 $\in \mathbb{Z}$

$$u = \langle \underset{\substack{\uparrow \\ \text{Schritt } j}}{k_j} \rangle = \sum_k k P_k \quad (4.4) \rightarrow \text{Driftbewegung für } u \neq 0!$$

(1) Beweis: $x_N = x_{N-1} + k_N L \quad (4.5)$

$$\rightarrow \langle x_N \rangle = \langle x_{N-1} \rangle + \langle k_N \rangle L = \langle x_{N-1} \rangle + uL$$

$$\rightarrow \boxed{\langle x_N \rangle = NuL} \quad (4.6)$$

Bsp: $P_{\pm 1} = \frac{1}{2} \rightarrow u = 0$

$$\begin{aligned}
(2) \text{ Var}(x_N) &= \langle (x_N - \langle x_N \rangle)^2 \rangle \\
&\stackrel{(4.5)}{=} \langle (x_{N-1} + k_N L - NuL)^2 \rangle \\
&\stackrel{(4.6)}{=} \langle [(x_{N-1} - \underbrace{(N-1)uL}_{\langle x_{N-1} \rangle}) + (k_N - u)L]^2 \rangle \\
&= \text{Var}(x_{N-1}) + \text{Var}(k_N)L^2 + 2 \underbrace{\langle (x_{N-1} - \langle x_{N-1} \rangle)(k_N - u)L \rangle}_{\langle k_N - u \rangle = 0} \\
&= NL^2 \text{Var}(k)
\end{aligned}$$

$$N = t/\Delta t \Rightarrow$$

$ \begin{aligned} \text{Var}(x_N) &= 2Dt, & D &= \frac{L^2}{2\Delta t} \text{Var}(k) \\ & & &= \frac{L^2}{2\Delta t} \langle (k-u)^2 \rangle \end{aligned} $	(4.7)
 universell	 Modell abhängig

Ausdehnung verschiedener Poymere

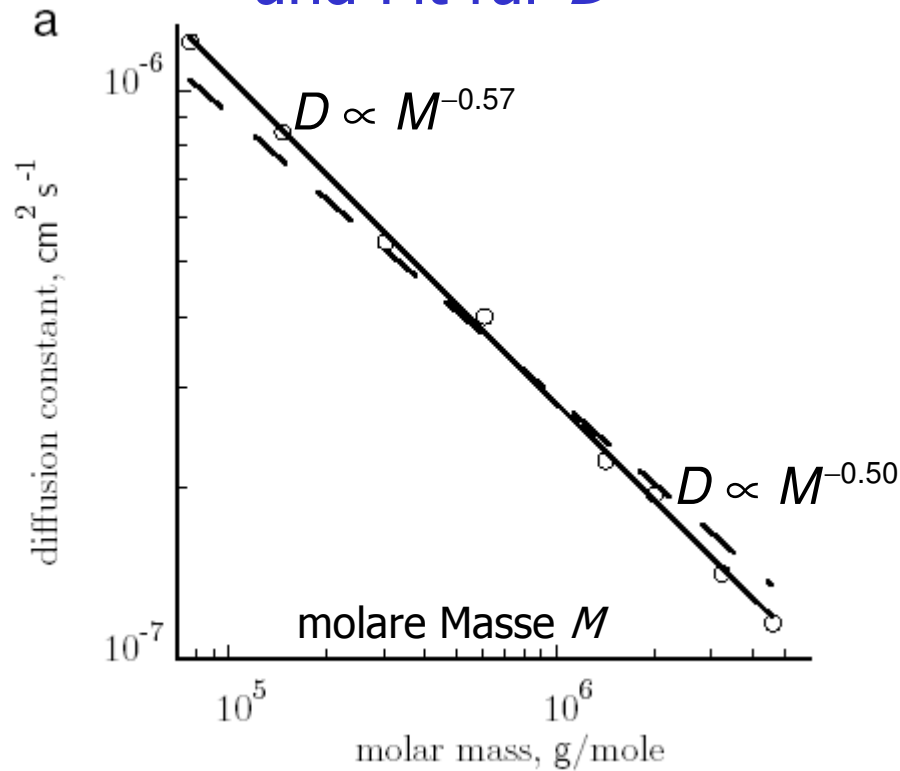
Table 4.1: [t:2A:coils]Properties of various polymers. The table shows the measured radius of gyration for a few natural and artificial polymers, along with the radius of the ball the polymer would occupy if it were tightly packed, estimated from the molar mass and approximate density. [From (Tanford, 1967)]

Polymer	Molar mass, g/mole	R_G , nm,	Packed-ball radius, nm	Type
Serum albumin	$6.6 \cdot 10^4$	3	2	compact
Catalase	$2.25 \cdot 10^5$	4	3	compact
Bushy stunt virus	$1.1 \cdot 10^7$	12	11	compact
Myosin	$4.93 \cdot 10^5$	47	4	extended
Polystyrene	$3.2 \cdot 10^6$	49	8	extended
DNA, in vitro	$4 \cdot 10^6$	117	7	extended

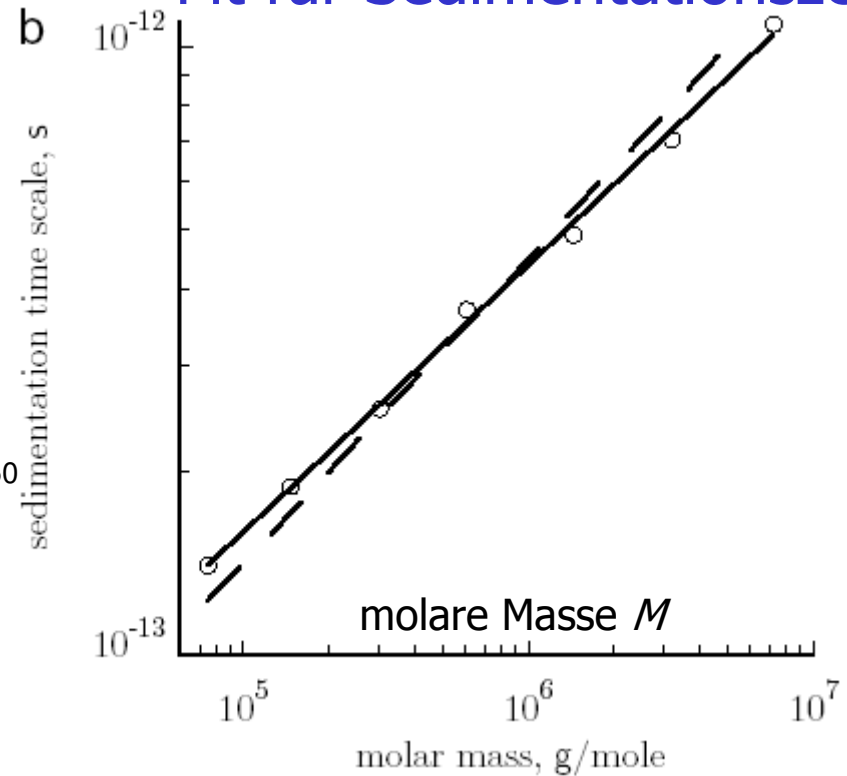
Gyrationsradius $R_G \propto \sqrt{\langle \mathbf{x}^2 \rangle}$

Radius einer dicht gepackten Kugel

Experimentelle Daten und Fit für D



Experimentelle Daten und Fit für Sedimentationszeit



4.7 (Experimental data with fits.) (a) Log-log plot of the diffusion constant D of polymethyl methacrylate in acetone, as a function of the polymer's molar mass M . The *solid line* corresponds to the function $D \propto M^{-0.57}$. For comparison, the *dashed line* shows the best fit with scaling exponent fixed to $-1/2$, which is the prediction of the simplified analysis in this chapter. (b) The sedimentation time scale s of the same polymer, to be discussed in Chapter 5. The solid line corresponds to the function $s \propto m^{0.44}$. For comparison, the dashed line shows the best fit with scaling exponent fixed to $1/2$. [Data from Meyerhoff & Schultz, 1952.]

2D-Zufallsweg von DNS

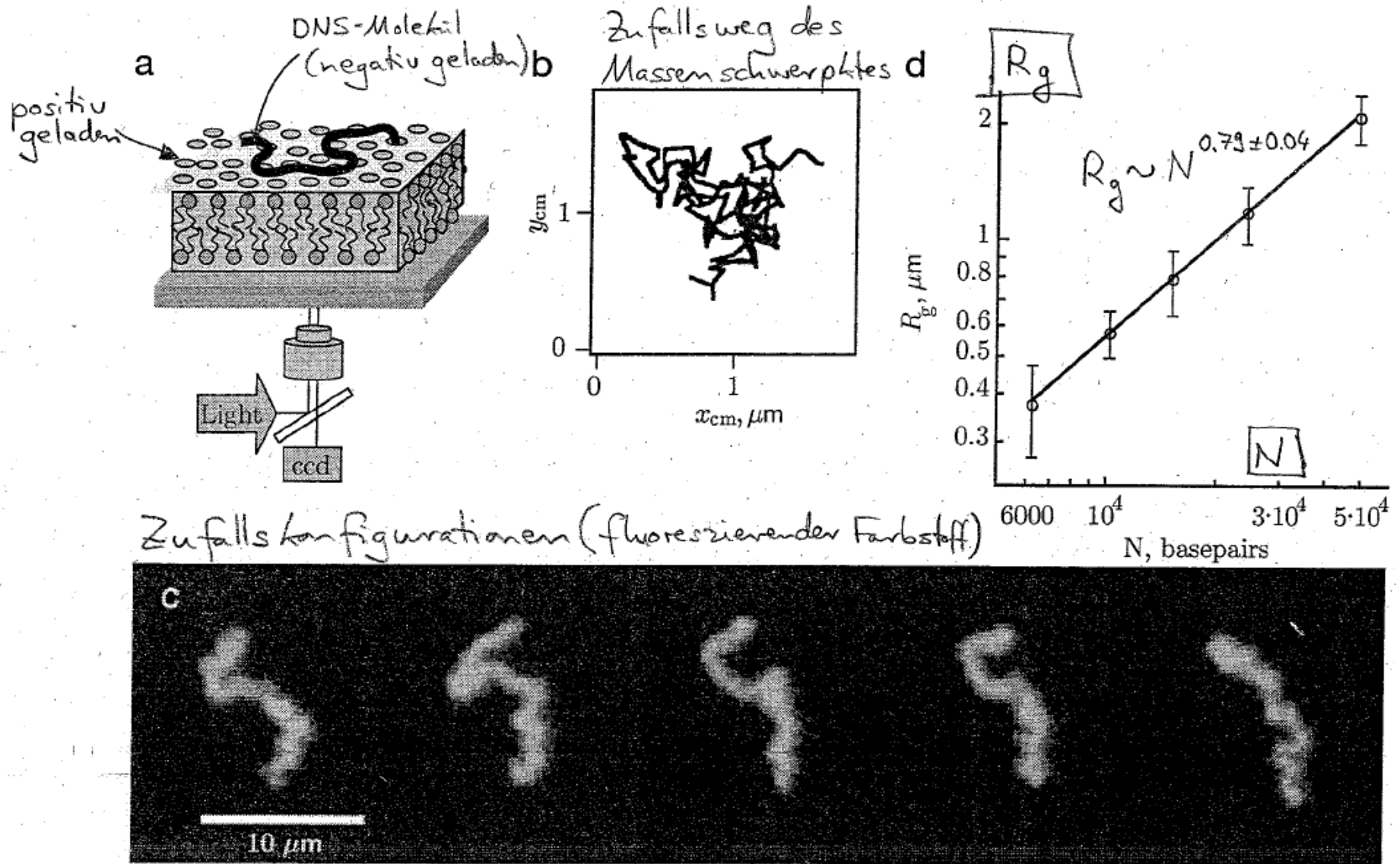


Figure 4.8:

Figure 4.8: (Schematic; experimental data; photomicrograph; experimental data.) Experimental test of the self-avoiding random-walk model of polymer conformation, in two dimensions. (a) Experimental setup. A negatively charged DNA molecule sticks to a positively charged surface. The DNA has been labeled with a fluorescent dye to make it visible in a light microscope. (b) The entire molecule performs a random walk in time. The plot shows its center of mass on successive observations (compare Figure 4.2 on page 103b,c). (c) Successive snapshots of the molecule taken at 2 s intervals. Each one shows a different random conformation. The fine structure of the conformation is not visible, due to the limited resolving power of an optical microscope, but the mean-square distance of the molecule from its center of mass can still be calculated. Scale bar, $10\ \mu\text{m}$. (d) Log-log plot of the size of a random coil of length N basepairs versus N . For each N , the coil size has been averaged over 30 independent snapshots like the ones in (c) (see Figure 4.5). The average size increases proportionally to $N^{0.79\pm 0.04}$, close to the theoretically predicted $N^{3/4}$ behavior (see Problem 7.9). [Digital image kindly supplied by B. Maier; see also (Maier & Rädler, 1999).]

Permeabilität \mathcal{P}_s

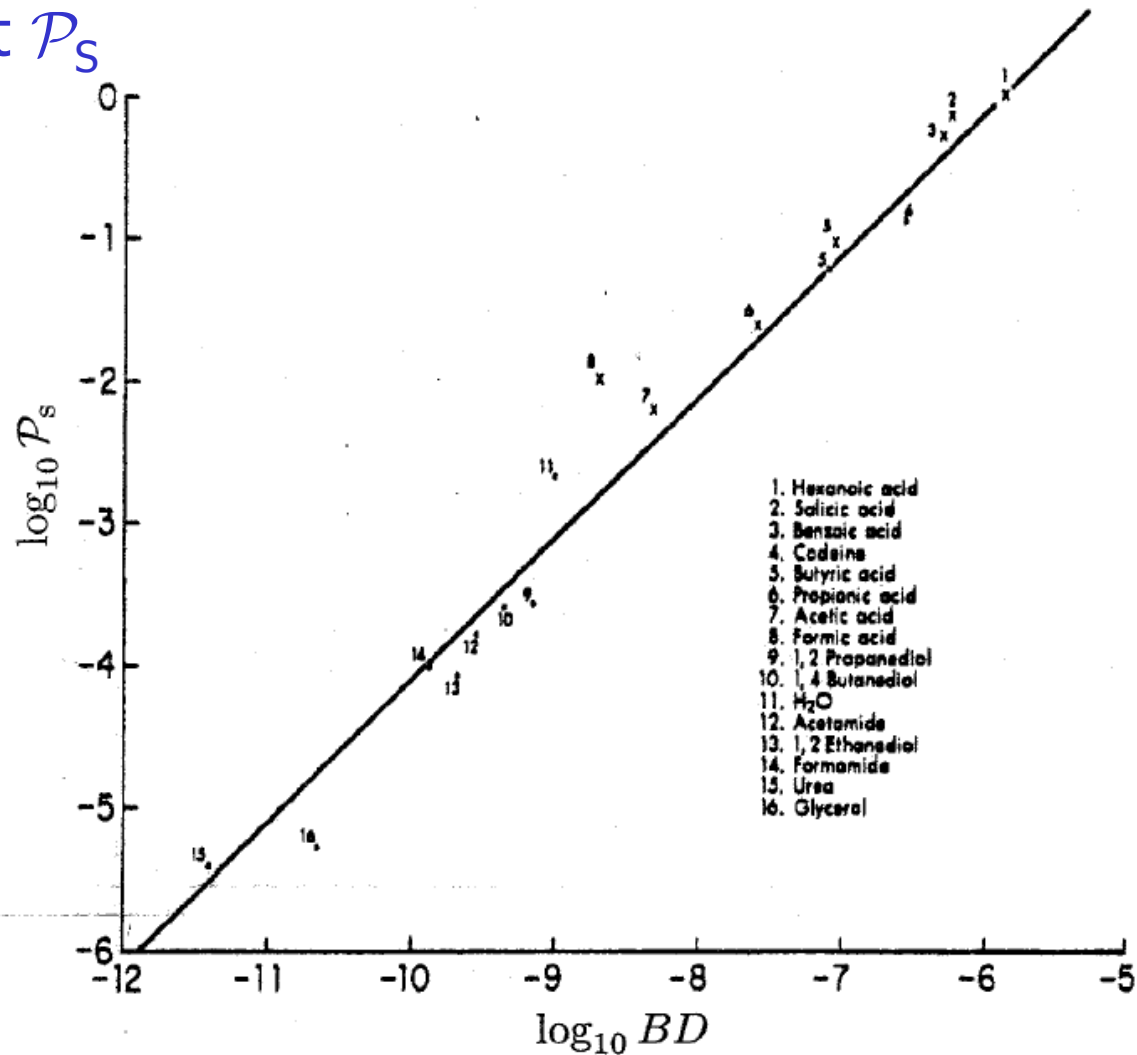


Figure 4.13: (Experimental data.) Log-log plot of the permeability \mathcal{P}_s of artificial bilayer membranes (made of egg phosphatidylcholine) to various solutes. The horizontal axis gives the product BD of the diffusion constant D of each solute in oil (hexadecane) times its partition coefficient B in oil versus water. \mathcal{P}_s is in cm/s, D in cm²/s, and B is dimensionless. The solid line has slope one, indicating a strict proportionality $\mathcal{P}_s \propto BD$. C. Overton pioneered experiments of this type in 1901. [Data from (Finkelstein, 1987).]