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6. Übungsblatt – Theoretische Festkörperphysik I,II

Abgabe: Fr. 04.06.2010 bis 12:00 Uhr, EW705.
Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Dreiergruppen erfolgen.

Aufgabe 14 (6 Punkte): *Free electrons in d dimensions*
Consider a gas of N free electrons in a d-dimensional hypervolume L^d. Calculate the density of states $\mathcal{D}_d(\epsilon)$ of the electrons from the dispersion relation $\epsilon = \hbar^2 k^2/2m$ and plot the result for the 1, 2, and 3 dimensions. Calculate the Fermi energy, Fermi wavenumber, and de Broglie wavelength at the Fermi surface, expressed in terms of the electron density $n = N/L^d$. Estimate these parameters for 1, 2, and 3 dimensional samples of GaAs.

Aufgabe 15 (7 Punkte): *Linearisation of the Boltzmann equation*

Write the electron distribution function of the Boltzmann equation as $f(\mathbf{r},\mathbf{p}) = f^{(0)}(\mathbf{r},\mathbf{p}) + f^{(1)}(\mathbf{r},\mathbf{p})$, where $f^{(0)}(\mathbf{r},\mathbf{p})$ is the Fermi-Dirac distribution and $f^{(1)}(\mathbf{r},\mathbf{p})$ is a first-order response function. Perform the appropriate linearisation of the Boltzmann equation

$$\frac{\partial}{\partial t} f(\mathbf{r},\mathbf{p},t) + \mathbf{v_p}\cdot\nabla_{\mathbf{r}} f(\mathbf{r},\mathbf{p},t) + e\left(\mathbf{E} + \mathbf{v_p}\times\mathbf{B}\right)\cdot\nabla_{\mathbf{p}} f(\mathbf{r},\mathbf{p},t) = I_{\mathbf{r},\mathbf{p},t}\left[f\right],$$

including a temperature gradient to obtain

$$\begin{aligned}\left(-\frac{\partial f^{(0)}(\mathbf{r},\mathbf{p})}{\partial\epsilon_p}\right)\mathbf{v_p}\cdot\left\{-\frac{\epsilon-\mu}{T}\nabla T + e\mathbf{E}\right\} &= -I_{\mathbf{r},\mathbf{p}}\left[f\right] + \mathbf{v_p}\cdot\nabla_{\mathbf{r}} f^{(1)}(\mathbf{r},\mathbf{p}) \\ &\quad + e\left(\mathbf{v_p}\times\mathbf{B}\right)\cdot\nabla_{\mathbf{p}} f^{(1)}(\mathbf{r},\mathbf{p}).\end{aligned}$$

Aufgabe 16 (7 Punkte): *Landau levels, Edge channels*

Consider a 2D gas of free electrons in the xy-plane in a perpendicular magnetic field with gauge chosen such that the vector potential is written $A_x = -By$, $A_y = A_z = 0$. Determine the eigenstates and energies of this system. Can these states carry a current?
The electrons are now confined in the y direction through the addition of potential $U(y)$. Assuming that this potential varies slowly relative to the y-variation of the above wavefunctions, use first-order pertubation theory to obtain an expression for the dispersion and group velocity of the electrons as a function of coordinate y. Use this result to explain the formation of edge channels in the quantum Hall effect.