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3. Excercise Sheet – Networks (with applications to neuroscience)

Due date: Fr. 24.06.2011 14:00

This excercise sheet is meant to provide help in order to recap the topics of this week's lectures. Solutions will be discussed during the Friday lecture.

Problem 6: *Reaction fronts (Schlögl model)*

1. Consider a reaction-diffusion system with a bistable dynamics according to

$$\frac{\partial u(x,t)}{\partial t} = f\big(u(x,t)\big) + D\frac{\partial^2 u(x,t)}{\partial x^2}$$

with $f\big(u(x,t)\big) = -k\,[u(x,t)-u_1]\,[u(x,t)-u_2]\,[u(x,t)-u_3]$ and $u_1 < u_2 < u_3$ and $k, D > 0$.
Show that the following profile is a solution of this equation:

$$u(\zeta) = \frac{u_1+u_3}{2} + \frac{u_1-u_3}{2}\tanh\left(\sqrt{\frac{k}{2D}}\frac{u_1-u_3}{2}\zeta\right)$$

with $\zeta = x - ct$ and a propagation velocity

$$c = \sqrt{\frac{kD}{2}}\,(u_1+u_3-2u_2)$$

for the transition from u_3 to u_1.

2. Verify the expression for the front width:

$$L = 4\sqrt{\frac{2D}{k}}\,(u_3-u_1)^{-1}$$

and sketch the soultion in the $\left(u, \frac{du}{d\zeta}\right)$ plane.

Problem 7: *Pulse solution in the FitzHugh-Nagumo model*
Consider a reaction-diffusion system based on the FitzHugh-Nagumo model:

$$\begin{aligned}\frac{\partial u(x,t)}{\partial t} &= 3u(x,t) - u(x,t)^3 - v(t) + \frac{\partial^2 u(x,t)}{\partial x^2}\\ \frac{\partial v(t)}{\partial t} &= \varepsilon[u(x,t)-\delta],\end{aligned}$$

where the parameter ε is small ($\varepsilon \ll 1$).

1. Draw the nullclines and analyze the stability of the fixed point.

2. In the limit $\varepsilon \to 0$ the inhibitor concentration v of the front stays constant. Verify the following expression for the pulse velocity:

$$c_0 = \sqrt{2}\left(-1.5\cos\frac{\varphi}{3} + 3\sin\frac{\pi}{3}\sin\frac{\varphi}{3}\right) \qquad \text{mit} \qquad \cos\varphi = \frac{v}{2}$$

and plot the function $c_0(v)$.
Hint: Use the Cardano formular for the calculation of the roots of a cubic polynomial and the results of the previous problem on the Schlögl model.

Problem 8: *Master stability function formalism*
The *master stability function* is a helpful tool to investigate the stability of a (completely) synchronous solution. It separates the local dynamics from the network topology.
Familiarize yourself with the main steps of the formalism as presented in the following paper:
L. M. Pecora and T. L. Carroll: *Master Stability Functions for Synchronized Coupled Systems*, Phys. Rev. Lett. **80**, 2109 (1998).