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1. Übungsblatt – TPVI: Quantensysteme im Nichtgleichgewicht

Abgabe: Fr. 27.04.2012 10:00-12:00, Uhr in der Vorlesung

Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Dreiergruppen erfolgen.

Aufgabe 1 (5 Punkte): *Unitarity of Scattering matrix*

Prove that the scattering matrix

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}$$

is unitary. Show that this implies the following relationships between the subblocks:

$$\begin{aligned} r^\dagger r + t^\dagger t &= r'^\dagger r' + t'^\dagger t' &= \mathbb{1} \\ r r^\dagger + t' t'^\dagger &= t t^\dagger + r' r'^\dagger &= \mathbb{1} \\ r^\dagger t' + t^\dagger r' &= t'^\dagger r + r'^\dagger t &= 0 \\ r t^\dagger + t' r'^\dagger &= t r^\dagger + r' t'^\dagger &= 0. \end{aligned}$$

Aufgabe 2 (8 Punkte): *Scatterers in series*

Consider two incoherent scatterers with transmission probabilities T_1 and T_2 .

- Show that the transmission probability of the two scatters in series is

$$T_{12} = \frac{T_1 T_2}{1 - R_1 R_2}; \quad \text{with } R_i = 1 - T_i.$$

- Using this result, or otherwise, show that the total transmission probability of N identical scatterers in series is

$$T(N) = \frac{T}{N(1 - T) + T}.$$

- Writing $N = \nu L$ where L is the sample length and ν the linear density of scatters, show that the transmission probability of length- L conductor can be written

$$T(L) = \frac{L_0}{L + L_0},$$

and find and interpret L_0 .

Bitte Rückseite beachten! →

1. Übung TPVI SSs12

Aufgabe 3 (7 Punkte): Quantum Point Contact

The transmission probability of the n th channel of QPC is approximately

$$T_n(E) = \theta(E - \epsilon_n),$$

with mode energy $\epsilon_n = n\hbar\omega_0$ from a harmonic confinement in the absence of magnetic field. Calculate, plot and discuss the conductance of the QPC at finite temperature.

A magnetic field is applied to the sample in the plane of the 2DEG such that the electrons acquire a Zeeman splitting but with orbital degrees of freedom unaffected. Again: calculate the conductance, plot and discuss.

Hint: use the following expression for the conductance :

$$G = \frac{e^2}{h} \sum_n \int dE T_n(E) \left(-\frac{\partial f(E - \mu)}{\partial E} \right).$$

Vorlesung:	<ul style="list-style-type: none">• Do. 10:00 Uhr – 12:00 Uhr im EW 203.• Fr. 10:00 Uhr – 12:00 Uhr im EW 203.
Übung:	<ul style="list-style-type: none">• Mi. 14–16 Uhr im EW 016 (Clive Emary).
Scheinkriterien:	<ul style="list-style-type: none">• Mindestens 50% der Übungspunkte.• Regelmäßige und aktive Teilnahme am Tutorium.• Schriftliche Arbeit.
Literatur:	<ul style="list-style-type: none">• M. Büttiker, Phys. Rev. B 46 12485 (1992).• Ya. M. Blanter, and M. Büttiker, <i>Shot Noise in Mesoscopic Conductors</i>, Phys. Rep. 336, 1 (2000); cond-mat/9910158.• H. Bruus and K. Flensberg, <i>Many-Body Quantum Theory in Condensed Matter Physics, An Introduction</i>, Oxford University Press, Oxford 2004.• S. Datta, <i>Electronic Transport in Mesoscopic Systems</i>, Cambridge University Press, Cambridge, 1995.