Prof. Dr. Tobias Brandes
Dr. Clive Emary

4. Übungsblatt – TPVI: Quantensysteme im Nichtgleichgewicht

Abgabe: Fr. 01.06.2012 10:00-12:00, Uhr in der Vorlesung
Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Dreiergruppen erfolgen.

Aufgabe 10 (12 Punkte): *FCS: Master equation approach*

A generic uni-directional master equation with single-particle tunneling may be written

$$\dot{\rho}(t) = \mathcal{W}\rho(t) = (\mathcal{W}_0 + \mathcal{J})\rho(t),$$

where ρ is the system density matrix, and $\mathcal{W}$ is the Liouvillian superoperator which consists of 'no-jump' and 'jump' parts, $\mathcal{W}_0$ and $\mathcal{J}$. The full density matrix can be split up into n-resolved components $\rho(t) = \sum_n \rho^{(n)}(t)$ where, in Laplace space,

$$\rho^{(n)}(z) \equiv \int_0^\infty dt e^{-zt}\rho^{(n)}(t) = \Omega_0(z)\left[\mathcal{J}\Omega_0(z)\right]^n \rho(t_0),$$

with $\Omega_0(z) = [z - \mathcal{W}_0]^{-1}$ and $\rho(t_0)$ the state of the system at time $t = t_0$.

(a) What is the physical meaning of $\rho^{(n)}(t)$?

(b) Show that $\rho^{(n)}(t)$ obeys the n-resolved master equation

$$\dot{\rho}^{(n)}(t) = \mathcal{W}_0\rho^{(n)}(t) + \mathcal{J}\rho^{(n-1)}(t);$$

(c) We introduce the counting field χ via the Fourier transform $\rho(\chi;t) = \sum_n e^{i\chi n}\rho^{(n)}(t)$. Find an explicit expression for $\rho(\chi;t)$ in terms of $\mathcal{W}(\chi) = \mathcal{W}_0 + e^{i\chi}\mathcal{J}$;

(d) Show that $\mathcal{G}(\chi;t) = \mathrm{Tr}[\rho(\chi;t)]$ is the moment generating function for charge transferred through the system in a time t;

(e) Show that, in the asymptotic limit $t \to \infty$, the cumulant generating function $\mathcal{F} = \log\mathcal{G}$ can be written as $\mathcal{F}(\chi;t) = \lambda_0(\chi)t$ with $\lambda_0(\chi)$ that eigenvalue of $\mathcal{W}(\chi)$ for which $\lambda_0(\chi \to 0) = 0$.

Bitte Rückseite beachten!⟶

Aufgabe 11 (8 Punkte): *FCS: Single resonant level*
Transport through a quantum dot with the Coulomb blockade and high-bias regimes can be described by the Liouvillian (with counting)

$$\mathcal{W}(\chi) = \begin{pmatrix} -\Gamma_L & \Gamma_R e^{i\chi} \\ \Gamma_L & -\Gamma_R \end{pmatrix},$$

with rates $\Gamma_{L,R}$.

(a) Calculate the asymptotic ($t \to \infty$) cumulant generating function for this model.

(b) Consider the limit $\Gamma_R \gg \Gamma_L$ and interpret your result.

(c) Calculate the probability distribution function $P(n;t) = \mathrm{Tr}[\rho^{(n)}(t)]$. Plot $P(n;t)$ as a function of n for increasing times and discuss. [Hint: this requires the inverse Fourier transform of the corresponding moment generating function. The requisite integral can be evaluated either numerically or with saddle-point method]

Vorlesung:	• Do. 10:00 Uhr – 12:00 Uhr im EW 203. • Fr. 10:00 Uhr – 12:00 Uhr im EW 203.
Übung:	• Mi. 14–16 Uhr im EW 016 (Clive Emary).
Scheinkriterien:	• Mindestens 50% der Übungspunkte. • Regelmäßige und aktive Teilnahme am Tutorium. • Schriftliche Arbeit.