

English summary

1.4 Small-world networks

- interpolate between regular ring and random network by rewiring
- small-world properties: (i) average shortest path length low
links between 2 nodes
- (ii) global cluster coefficient high
links between neighbors
possible links
- application to robustness w.r.t. spreading processes
→ (average) shortest path $\hat{=}$ worst case scenario / infection route
→ random network structure facilitates spreading due to shortcuts

2 Meta population models of disease spreading

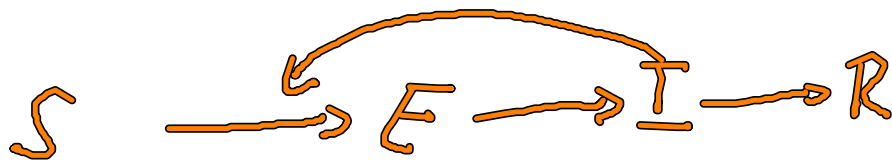
2.1 SIR model (See lecture 2 & 3)

- derivation of $\dot{S} = -\beta SI$ assuming homogeneous mixing
fraction of susceptibles
"frequency dependent" (cf. $\dot{X} = -\beta XY$: "density dependent")
susceptibles

$$\dot{S} = -\beta SI, \quad \dot{I} = \beta SI - \gamma I, \quad \dot{R} = \gamma I, \quad 1 = R + S + I$$

• asymptotics & solutions: $S(t) = S(0) e^{-R_0 R(t)}$
 $R(t) = \frac{1}{R_0 S_0} \left[S_0 R_0 - 1 + \sigma \tanh\left(\frac{1}{2} \sigma \mu t + \phi\right) \right]$

2.2 SEIR-Modell



↑ "exposed" (schon ansteckend, aber noch nicht infektiös)
 \Rightarrow latente Periode: $\frac{1}{\sigma}$

Modellgleichung: $\dot{S} = \mu - (\beta I + \mu) S$ $1 = S + E + I + R$

$$\dot{E} = \beta S I - (\mu + \sigma) E$$

$$\dot{I} = \sigma E - (\mu + \gamma) I$$

$$\dot{R} = \gamma I - \mu R$$

Berechne Fixpunkte: (i) krankheitsfrei: $S^* = 1$, $E^* = I^* = R^* = 0$

(ii) endemisch: $S^* = \frac{1}{R_0}$ mit $R = \frac{(\beta + \mu)(\mu + \sigma)}{\beta \sigma}$

$$\Rightarrow E^* = \frac{\mu(\mu + \sigma)}{\beta \sigma} (R_0 - 1)$$

$$\Rightarrow I^* = \frac{\mu}{\beta} (R_0 - 1)$$

\Rightarrow Berechne Eigenwerte der Jacobi-Matrix zur (linearen)

Stabilitätsanalyse: Eigenwerte einer 4×4 Matrix

\Rightarrow Lösen der charakteristischen Gleichung (4. Ordnung)

Annahme $\nabla, \sigma \Rightarrow \mu, \mu R_0 \Rightarrow$ Faktorisierung des charakteristischen Polynoms

\Rightarrow quadratische Gleichung

Auflösung am 10.6.

Siehe auch SIS-Modell (ggf mit Adaptionen)

\hookrightarrow TK-Seite: Direktzugriff 101629