PD Dr. Gernot Schaller
Dr. Javier Cerrillo

10. Übungsblatt – TPVI: Quantensysteme im Nichtgleichgewicht II

Abgabe: Mi. 12.07.2017 12:15 Uhr im Tutorium

Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Zweiergruppen erfolgen.

Aufgabe 12 (20 Punkte): *Landauer's Principle*

Landauer's principle states that the heat dissipated in the erasure of information $\langle Q \rangle_t$ is at least equal to the corresponding entropy change $\Delta S(t)/\beta$

$$\beta \langle Q \rangle_t \geq \Delta S(t). \tag{1}$$

Its role in the resolution of Maxwell's daemon paradox is well known. A recent work (arXiv:1704.01078) provides a tighter bound to the inequality (1) based on the formalism of full counting statistics.

(a) (10) Given the definition of the cumulant generating function of the heat transfer Q under initial bath temperature β

$$\Theta(\eta, \beta, t) \equiv \ln \left\langle e^{-\eta Q} \right\rangle_t = \ln \int p_t(Q) e^{-\eta Q} dQ, \tag{2}$$

as the Laplace transform of the corresponding probability distribution $p_t(Q)$, show that it is a convex function with respect to the counting parameter η

$$\Theta(\eta, \beta, t) \geq \eta \frac{\partial}{\partial \eta} \Theta(\eta, \beta, t)|_{\eta=0}. \tag{3}$$

For this, use Jenssen's inequality, which states that, given a convex function ϕ and a random variable X,

$$\phi\left(\langle X \rangle\right) \leq \langle \phi(X) \rangle. \tag{4}$$

(b) (5) Use the above result to define a one-parameter family of lower and upper bounds $L(\eta)$ and $U(\eta)$ to the disspated heat

$$U(\eta) \geq \beta \langle Q \rangle_t \geq L(\eta). \tag{5}$$

(c) (5) Show that the bounds are tight in the limit $|\eta| \to 0$ unless a critical point exists at $\eta = 0$.