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2. Übungsblatt – TPVI: Quantensysteme im Nichtgleichgewicht II

Abgabe: Mi. 10.05.2017 12:15 Uhr im Tutorium

Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Zweiergruppen erfolgen.

Aufgabe 3 (15 Punkte): *Born-Markov-secular (BMS) master equation*

Let us consider a diagonalization of the system Hamiltonian $\mathcal{H}_S = \sum_a E_a |a\rangle\langle a|$ and a decomposition of the interaction Hamiltonian in a sum of Hermitian operators $\mathcal{H}_I = \sum_\alpha A_\alpha B_\alpha$. The correlation function of the bath is defined as

$$C_{\alpha\beta}(\tau) = \mathrm{tr}\left(e^{i\mathcal{H}_B\tau} B_\alpha e^{-i\mathcal{H}_B\tau} B_\beta \bar{\rho}_B\right) \tag{1}$$

and its even and odd Fourier transforms are $\gamma_{\alpha\beta}(\omega)$ and $\sigma_{\alpha\beta}(\omega)$ respectively. Further definitions can be found in section 2.2.1 and 2.2.2 of the lecture notes.

(a) (5) Show that the Lamb-shift Hamiltonian H_{LS}

$$H_{LS} = \sum_{\alpha\beta} \sum_{a,b,c} \frac{1}{2i} \sigma_{\alpha\beta}(E_b - E_c) \delta_{E_b,E_a} A_\beta^{cb} \left(A_\alpha^{ca}\right)^* |a\rangle\langle b|, \tag{2}$$

is Hermitian and commutes with the system Hamiltonian.

(b) (5) Show the validity of the Kubo-Martin-Schwinger condition

$$C_{\alpha\beta}(\tau) = C_{\beta\alpha}(-\tau - i\beta) \tag{3}$$

for a thermal bath with $\bar{\rho}_B = \frac{e^{-\beta\mathcal{H}_B}}{\mathrm{tr}\left(e^{-\beta\mathcal{H}_B}\right)}$.

(c) (5) Show that $\bar{\rho}_S = \frac{e^{-\beta\mathcal{H}_S}}{\mathrm{tr}\left(e^{-\beta\mathcal{H}_S}\right)}$ is a stationary state of the BMS master equation when $\gamma_{\alpha\beta}(-\omega) = \gamma_{\beta\alpha}(\omega) e^{-\beta\omega}$.

Aufgabe 4 (10 Punkte): *Coarse-graining (CG) master equation*

Definitions of the symbols used in this exercise can be found in section 2.2.3 of the lecture notes.

(a) (5) By assuming Hermitian coupling operators $A_\alpha = A_\alpha^\dagger$, show that the CG master equation is of Lindblad form for all coarse-graining times τ.

(b) (5) (CG-BMS correspondence) Show for Hermitian coupling operators that when $\tau \to \infty$, CG and BMS approximation are equivalent. You may use the identity

$$\lim_{\tau\to\infty} \tau \mathrm{sinc}\left[\frac{\tau}{2}\left(\Omega_a - \omega\right)\right] \mathrm{sinc}\left[\frac{\tau}{2}\left(\Omega_b - \omega\right)\right] = 2\pi \delta_{\Omega_a,\Omega_b} \delta\left(\Omega_a - \omega\right). \tag{4}$$