PD Dr. Gernot Schaller
Dr. Javier Cerrillo

# 3. Übungsblatt – TPVI: Quantensysteme im Nichtgleichgewicht II

**Abgabe: Mi. 17.05.2017 12:15 Uhr im Tutorium**

*Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Zweiergruppen erfolgen.*

**Aufgabe 5 (20 Punkte):** *Spohn's inequality*

The quantum relative entropy defined as

$$D(\rho||\sigma) = \mathrm{tr}\left(\rho\left(\ln\rho - \ln\sigma\right)\right), \tag{1}$$

is contractive under the effect of a Kraus map $\mathcal{K}$

$$D(\mathcal{K}\rho||\mathcal{K}\sigma) \leq D(\rho||\sigma). \tag{2}$$

(a) (5) Using Eq.(2), derive a general expression of Spohn's inequality and the specific form for a system connected to two terminals with particle and heat exchange:

$$\dot{S}_i = \dot{S} - \beta_L\left(I_E^{(L)} - \mu_L I_M^{(L)}\right) - \beta_R\left(I_E^{(R)} - \mu_R I_M^{(R)}\right) \geq 0. \tag{3}$$

(b) (5) For the conventions $\beta_L < \beta_R$ and $\mu_L < \mu_R$, calculate the upper bound for the coefficient of performance for heating

$$\mathrm{COP}_{\mathrm{heating}} = \frac{\dot{Q}_{\mathrm{hot}}}{\dot{W}_{\mathrm{cons}}} \leq \frac{T_{\mathrm{hot}}}{T_{\mathrm{hot}} - T_{\mathrm{cold}}}. \tag{4}$$

(c) (5) For the single electron transistor, we have that $I_E^{(R,L)} = \epsilon I_M^{(R,L)}$, where $\epsilon$ is the energy level of the dot. Given a temperature bias $\Delta\beta = \beta_R - \beta_L$, use Eq.(3) to determine the voltage $V$ (such that $\mu_R = \frac{V}{2}$ and $\mu_L = -\frac{V}{2}$) that results in vanishing stationary currents.

(d) (5) Given the rate equation for the single-electron transistor

$$\frac{d}{dt}\begin{pmatrix} P_0 \\ P_1 \end{pmatrix} = \mathcal{W}\begin{pmatrix} P_0 \\ P_1 \end{pmatrix} = \begin{pmatrix} -\Gamma_L f_L - \Gamma_R f_R & \Gamma_L(1-f_L) + \Gamma_R(1-f_R) \\ \Gamma_L f_L + \Gamma_R f_R & -\Gamma_L(1-f_L) - \Gamma_R(1-f_R) \end{pmatrix}\begin{pmatrix} P_0 \\ P_1 \end{pmatrix},$$

compute the stationary particle current between the left and the right reservoirs by means of the condition of stationarity

$$I_M^{(R)} = -I_M^{(L)}. \tag{5}$$