PD Dr. Gernot Schaller
Dr. Javier Cerrillo

5. Übungsblatt – TPVI: Quantensysteme im Nichtgleichgewicht II

Abgabe: Mi. 31.05.2017 12:15 Uhr im Tutorium

Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Zweiergruppen erfolgen.

Aufgabe 7 (20 Punkte): *Cumulant generating functions*

It is possible to characterize the statistics of the measurement of an observable O by means of the associated cumulant generating function

(1)
$$G(\chi, \beta_\nu; t) = \ln \operatorname{tr}\left(e^{i\chi O} \rho_T(t)\right)$$

where $\rho_T(t)$ is the density matrix of the whole system, including the environment. Let us assume an initial thermal state $\rho_\nu = \frac{e^{-\beta_\nu H_\nu}}{\operatorname{tr}\left(e^{-\beta_\nu H_\nu}\right)}$ for the subspace ν where the measurement is performed. Additionally, $[O, H_\nu] = 0$.

(a) (5) In the steady-state limit $\lim_{t\to\infty} G(\chi, \beta_\nu; t) \equiv G(\chi)$, time inversion symmetry establishes for certain types of cumulant generating functions a fluctuation theorem of the form

(2)
$$G(\chi) = G(-\chi + i\beta_\nu).$$

Use the theorem to derive steady-state fluctuation-dissipation theorems

(3)
$$L_m^n = \sum_{j=0}^{m} \left(\begin{array}{c} m \\ j \end{array} \right) (-1)^{n+j} L_{m-j}^{n+j},$$

where $L_m^n = \frac{d^m}{d\beta_\nu^m} \langle O^n \rangle$. What form does the expression take for $n = m = 1$?

(b) (10) Show that the cumulant generating function $G_T(\chi, t)$ associated to the difference in outcomes of measurements of operator O at time t and $t = 0$ has the form

(4)
$$G_T(\chi, \beta_\nu; t) = \ln \operatorname{tr}\left(U^\dagger(t) e^{i\chi O} U(t) e^{-i\chi O} \rho_T(0)\right),$$

where $U(t)$ is the evolution operator of the full system.

(c) (5) Confirm that, for $O = H_\nu$, a similar symmetry as the fluctuation theorem Eq.(2) relates functions G and G_T

(5)
$$G(\chi, \beta_\nu; t) - G(\chi, \beta_\nu; 0) = G_T(\chi, \beta_\nu - i\chi; t).$$

Show that this leads to the relations

(6)
$$L_m^n(t) - L_m^n(0) = \sum_{j=0}^{n} \left(\begin{array}{c} n \\ j \end{array} \right) (-1)^j L_{T,m+j}^{n-j}(t),$$

where $L_{T,m}^n(t) = \frac{d^m}{d\beta_\nu^m} \frac{\partial^n}{\partial(i\chi)^n} \left. G_T(\chi, \beta_\nu; t)\right|_{\chi=0}$.