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7. Übungsblatt – TPVI: Quantensysteme im Nichtgleichgewicht II

Abgabe: Mi. 14.06.2017 12:15 Uhr im Tutorium

Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Zweiergruppen erfolgen.

Aufgabe 9 (20 Punkte): *Feedback control of quantum transport*

As explored by Prof. Brandes in *Phys. Rev. Lett.* **105**, 060602 (2010), it is possible to control the statistics of quantum transport by means of measurement and feedback. In particular, it is possible to freeze the growth of noise, thereby establishing a means for regular sources of intensity. For the paradigmatic example of unidirectional transport through a tunnel junction with no internal degrees of freedom, the master equation conditioned to the number n of tunneled electrons has the form

$$\dot{\rho}^{(n)}(t) = \mathcal{L}^0_{(n)}(t)\rho^{(n)}(t) + \mathcal{J}_{(n-1)}(t)\rho^{(n-1)}(t), \tag{1}$$

where the jump and Liouville coefficients $\mathcal{L}^0_{(n)}(t) = -\mathcal{J}_{(n)}(t) = -\Gamma f(q_n(t))$ have been made dependent on the measurement outcome n by means of the auxiliary function $f(q_n(t))$, with

$$q_n(t) \equiv I_0 t - n, \tag{2}$$

and $f(0) = 1$. This modulates the bare tunneling rate of electrons through the junctions Γ multiplicatively proportionally to the deviation of the transported charge n with respect to the average value without feedback $I_0 t$. Let us consider linear feedback, i.e. $f(x) = 1 + gx$, where the positive feedback strength $g \ll 1$ compensates large charge fluctuations by a modulation of the average current I_0.

(a) (5) Calculate the average current with feedback

$$C_1(t) \equiv \langle n \rangle_t = \Gamma t, \tag{3}$$

and the variance

$$C_2(t) \equiv \langle n^2 \rangle_t - \langle n \rangle_t^2 = \frac{1}{2g}\left(1 - e^{-2g\Gamma t}\right), \tag{4}$$

directly from their definitions.

(b) (5) By using the definition $\rho(\chi, t) \equiv \sum_n \rho^{(n)}(t) e^{i\chi n}$, transform the master equation (1) into the partial differential equation

$$\frac{\partial}{\partial t}\rho(\chi, t) = \mathcal{L}(\chi, t) f\left(I_0 t - \frac{\partial}{\partial i\chi}\right)\rho(\chi, t) \tag{5}$$

and find the form of $\mathcal{L}(\chi, t)$.

(c) (5) Show that the cumulant generating function $\mathcal{F}(\chi, t) \equiv \ln \rho(\chi, t)$ has the form

$$\mathcal{F}(\chi, t) = I_0 t i\chi + \frac{1}{g}\ln\left[e^{i\chi}\left(1 - e^{-g\Gamma t}\right) + e^{-g\Gamma t}\right] + \frac{1}{g}\left[Li_2\left((1 - e^{-i\chi})e^{-g\Gamma t}\right) - Li_2\left(1 - e^{-i\chi}\right)\right],$$

where $Li_2(z) \equiv \int_z^0 \frac{dt}{t}\ln(1 - t)$.

(d) (5) Show that the long-time form of $k \geq 2$ cumulants is

$$C_k\left(t \to \infty\right) = -\frac{1}{g}B_{k-1} \tag{6}$$

where the $B_k \equiv \left.\frac{d^k}{dx^k}\frac{x}{e^x - 1}\right|_{x=0}$ are the kth Bernoulli-Seki numbers.