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8. Übungsblatt – TPVI: Quantensysteme im Nichtgleichgewicht II

Abgabe: Mi. 28.06.2017 12:15 Uhr im Tutorium

Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Zweiergruppen erfolgen.

Aufgabe 10 (20 Punkte): *Feedback current rectification in single-electron-transistor*

Feedback mechanisms can be implemented to induce a current against a bias. Let us regard the case of a feedback-controlled single-electron-transistor, where a quantum point contact measures the presence of an electron in the quantum dot and modifies the tunneling rates based on the results empty (E) or full (F). The corresponding master equation reads

$$\frac{d}{dt}\begin{pmatrix} P_E \\ P_F \end{pmatrix} = \begin{pmatrix} -\Gamma_L^E f_L - \Gamma_R^E f_R & \Gamma_L^F (1-f_L)\, e^{i\chi_L} + \Gamma_R^F (1-f_R)\, e^{i\chi_R} \\ \Gamma_L^E f_L e^{-i\chi_L} + \Gamma_R^E f_R e^{-i\chi_R} & -\Gamma_L^F (1-f_L) - \Gamma_R^F (1-f_R) \end{pmatrix} \begin{pmatrix} P_E \\ P_F \end{pmatrix},$$

where $f_{R,L}$ correspond to the Fermi functions associated to the left (L) or right (R) leads, $\Gamma_{R,L}^{F,E}$ the tunnelling rates conditioned on the measurement outcome and $\chi_{R,L}$ the particle counting fields associated to each lead.

(a) (5) Calculate the average current with feedback in the steady state for zero bias $f_L = f_R = f$ as a function of f. For this, use $P_E = 1 - P_F$ and the condition $\frac{d}{dt} P_F = 0$ to find the steady state distribution in the dot. Use then this expression on the counting-field derivative of the master equation. What happens at zero temperatures, where $f \to \{0, 1\}$?

(b) (10) Let us parametrize the effect of the feedback in terms of an exponential suppression/enhancement δ of the tunnelling rates of the form

$$\begin{aligned} \Gamma_R^E &= \Gamma_L^F = e^{\delta}\Gamma, \\ \Gamma_R^F &= \Gamma_L^E = e^{-\delta}\Gamma. \end{aligned} \tag{1}$$

In addition, let us consider the situation of equal temperatures β but finite bias V. The statistics of the total current going through the dot is characterized by one of the counting fields, so let us assume $\chi_R = \chi$ and $\chi_L = 0$. Prove the symmetry of the cumulant generating function

$$\mathcal{F}(-\chi) = \mathcal{F}(\chi + i(4\delta + \beta V)) \tag{2}$$

by showing that the eigenvalues of the Liouvillian satisfy the same symmetry. Note that $\beta V = \ln \frac{f_L}{1-f_L} - \ln \frac{f_L}{1-f_L}$.

(c) (5) Derive from (2) the alternative form of the fluctuation theorem

$$\lim_{t\to\infty} \frac{P_n(t)}{P_{-n}(t)} = e^{n\beta(V-V^*)} \tag{3}$$

Show for equal temperatures that the feedback current vanishes when $V = V^* = -\delta/\beta$.