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9. Übungsblatt – TPVI: Quantensysteme im Nichtgleichgewicht II

Abgabe: Mi. 05.07.2017 12:15 Uhr im Tutorium

Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Zweiergruppen erfolgen.

Aufgabe 11 (20 Punkte): *Polaron-Transformed Master Equation*

Let us consider the interaction of a single electron transistor (annihilator d for the central dot and $c_{k\nu}$ for the leads) with Q phonon modes (annihilators a_q), so that the Hamiltonian of the full system reads

$$H = \left[\epsilon + \sum_{q=1}^{Q} \left(h_q a_q + h_q^* a_q^\dagger\right)\right] d^\dagger d + \sum_{\nu\in\{L,R\}} \sum_k \left(\epsilon_{k\nu} c_{k\nu}^\dagger c_{k\nu} + t_{k\nu} d c_{k\nu}^\dagger + t_{k\nu}^* c_{k\nu} d^\dagger\right) + \sum_{q=1}^{Q} \omega_q a_q^\dagger a_q,$$

with ϵ, $\epsilon_{k\nu}$ and ω_q the energies of the respective modes and h_q, $t_{k\nu}$ their interaction strengths.

(a) (10) In order to treat the non-equilibrium dynamics, a polaron transformation is introduced with the unitary

$$U = \exp\left[d^\dagger d \sum_q \left(\frac{h_q^*}{\omega_q} a_q^\dagger - \frac{h_q}{\omega_q} a_q\right)\right] \equiv e^{d^\dagger d X}$$

Show the effect of this transformation on the operators d, $d^\dagger$, a_q and $a_q^\dagger$ and the form of the transformed Hamiltonian

$$H' = \left(\epsilon + \sum_{q=1}^{Q} \frac{|h_q|^2}{\omega_q}\right) d^\dagger d + \sum_{\nu\in\{L,R\}} \sum_k \left(\epsilon_{k\nu} c_{k\nu}^\dagger c_{k\nu} + t_{k\nu} d c_{k\nu}^\dagger e^{-X} + t_{k\nu}^* c_{k\nu} d^\dagger e^{X}\right) + \sum_{q=1}^{Q} \omega_q a_q^\dagger a_q.$$

(b) (5) The relevant correlation function for the new interaction terms

$$\sum_{\nu\in\{L,R\}} \sum_k t_{k\nu} d c_{k\nu}^\dagger e^{-X},$$

and its Hermitian conjugate, can be split into electronic and phonon contributions $C_{12}^{\nu,\chi}(\tau) = C_{12,el}^{\nu}(\tau) C_{12,ph}^{\chi}(\tau)$, where the counting field χ is the conjugate of the number operator $N = \sum_q a_q^\dagger a_q$. Show that the phonon correlation function

$$(1) C_{12,ph}^{\chi}(\tau) = \exp\left\{\sum_q \frac{|h_q|^2}{\omega_q^2} \left[e^{-i(\omega_q\tau-\chi)}\left(1+n_B^q\right) + e^{i(\omega_q\tau-\chi)} n_B^q - \left(1+2n_B^q\right)\right]\right\}.$$

with $n_B^q = \left(e^{\beta_{ph}\omega_q} - 1\right)^{-1}$, obeys the KMS condition

$$(2) \qquad C_{12,ph}^{\chi}(\tau) = C_{12,ph}^{\chi}(-\tau - i\beta_{ph}).$$

Bitte Rückseite beachten!⟶

(c) (5) For the simplification $Q = 1$, the correlation function (1) may be decomposed into $C^{\chi}_{12,ph}(\tau) = \sum_n C^n_{12,ph} e^{-in(\omega_q \tau - \chi)}$. The correlation function of the electronic leads has the form $\gamma^{\nu}_{12,el}(\omega) = \Gamma_{\nu}(\omega) f_{\nu}(\omega)$, with $\Gamma_{\nu}(\omega) = \sum_k |t_{k\nu}|^2 \delta(\omega - \epsilon_{k\nu})$ and $f_{\nu}(\omega)$ the Fermi function for inverse temperature β_{ν} and chemical potential μ_{ν}. Show that the correlation function of the interaction in frequency space for n transfered phonons $\gamma^{\nu}_{12,n}(\omega) = \gamma^{\nu}_{12,el}(\omega - n\omega_q) C^n_{12,ph}$ obeys the KMS relation

$$\gamma^{\nu}_{12,n}(-\omega) = \gamma^{\nu}_{12,-n}(\omega) e^{-\beta_{\nu}(\omega - \mu_{\nu} + n\omega_q)} e^{\beta_{ph} n \omega_q}. \tag{3}$$