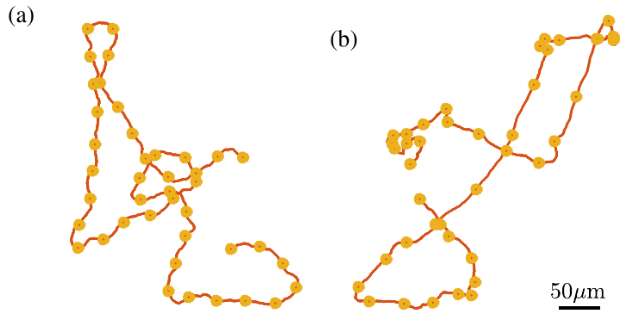
Active Brownian Particle VS Active Ornstein-Uhlenbeck Particle

Two Models For One System?

The capability of converting energy from surrounding environment into a directed motion derives actives particles far from equilibrium. This directed motion destroys the time-reversibility at the microscopic length-scales and allows the active particles (AP) to exhibit behaviors which are impossible in thermal equilibrium systems. We will consider two different mathematical models of APs and compare the characteristic quantities calculated from these models. Comparing these quantities we will be able to check when these models are equivalent and how different parameters in both models are connected. Within this project, we will learn, how the standard techniques in stochastic processes could be utilized to explain complicated observations in the nature and laboratory.

I. PROJECT DESCRIPTION

Active Brownian particle is a mathematical model for a class of self-propelled particles which swim at fixed speed along a director vector and it rotates by a slow rotational diffusion. Self-phoretic colloids such as Janus particle belong to this class of APs. Another model for describing self-propelled particles is the active Ornstein-Uhlenbeck particle. In this model, the particle swims with a velocity vector which each Cartesian component of the velocity follows the standard Ornstein-Uhlenbeck process. Therefore the velocity is damping as the time passes until the next kick due to the thermal noise acts.



Rotational diffusion dynamic **Gaussian noise dynamic**

Are these two models equivalent? At which time scales do we expect to observe exactly the same behavior from both models? Which parameters in these models are connected? To address these questions, we are required to analytically study the characteristic quantities which thoroughly describe these models and compare them. In the following, we list theses quantities which have to calculated for both models.

II. STEPS TO BE MADE

• Step zero: Write down the Langevin equation of motion for your model.

Here the properties of the stochastic forces in the equation determines to a large extent the dynamic of the system.

• Step one: Calculate the mean position of the particle using direct integration of the stochastic differential equation of motion.

This calculation allows us to gain information about the persistent motion of AP.

• Step two: Calculate the mean squared displacement(MSD) of the particle using direct integration of the stochastic differential equations. Plot the MSD.

Results of this step would make the calculation of the diffusion coefficient possible. Here, one would be able to determine the timescales at which the MSD undergoes different crossovers and therefore calculate the limiting diffusion coefficients at short and long time asymptotics.

• Step three: Derive the Fokker-Plank equation for the probability distribution function (PDF) of particle displacement corresponding to stochastic differential equation of motion.

Here we will learn how the deterministic description of the motion could be translated to the probabilistic analysis.

• Step four: Solve the Fokker-plank equations (applying the needed approximations). Plot the PDF at different time scales.

The transition of the PDF from non-Gaussian to Gaussian form could be tracked as the time becomes longer than specific characteristic time.

• Step five: Repeat steps one to three using the PDF in calculations instead of direct integration of equation of motion.

• Step six: (Optional) Run some Brownian dynamic simulations and compare the results with the theory for MSD and PDF.

In this step we can confirm our results in previous steps using another approach.

III. RELEVANT LITERATURE

∗ Review article: Bechinger, C., Di Leonardo, R., Lwen, H., Reichhardt, C., Volpe, G., and Volpe, G. (2016). Active particles in complex and crowded environments. Reviews of Modern Physics, 88(4), 045006.

∗ Review article: Romanczuk, P., Br, M., Ebeling, W., Lindner, B., and Schimansky-Geier, L. (2012). Active Brownian Particles-From Individual to Collective Stochastic Dynamics p. The European Physical Journal Special Topics, 202.

∗ Standard books or lecture notes for stochastic processes.

Suggestion: https://courses.physics.ucsd.edu/2015/Fall/physics210b/LECTURES/CH02.pdf

∗ Sevilla, F. J., and Sandoval, M. (2015). Smoluchowski diffusion equation for active Brownian swimmers. Physical Review E, 91(5), 052150.

∗ Lindner, B., and Nicola, E. M. (2008). Diffusion in different models of active Brownian motion. The European Physical Journal Special Topics, 157(1), 43-52.

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