Prof. Dr. Eckehard Schöll, PhD und Dr. Kathy Lüdge
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3. Übungsblatt – Theoretische Physik VI: Nichtgleichgewichtsstatistik

Abgabe: Mo. 15.11.2010 in der Übung

Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Dreiergruppen erfolgen.

Aufgabe 4 (5 Punkte): *Diffusion in a gravitational field*

A strongly damped Brownian particle moving in a constant gravitational field (strength g) is described by the Fokker-Planck equation

$$\frac{\partial}{\partial t}p = \frac{\partial}{\partial x}(gp) + D\frac{\partial^2}{\partial x^2}p$$

Solve for stationary distribution $p^*(x)$ on the interval (a, b) with reflecting boundaries. What conditions does normalisability place on a and b? By comparing your solution with what you know from $equilibrium$ statistical physics, relate the diffusion constant D to temperature.

Aufgabe 5 (4 Punkte): *Wiener process with absorbing boundaries*

By making an appropriate series Ansatz, solve Fokker-Planck equation $\frac{\partial}{\partial t}p = D\frac{\partial^2}{\partial x^2}p$ for $p(x,t)$ on the interval $(0, 1)$ with absorbing boundary conditions such that $p(0,t) = p(1,t) = 0$. What would be the analogous conditions for reflecting boundaries and what Ansatz would one make in that case?

Aufgabe 6 (6 Punkte): *Ornstein-Uhlenbeck Process*

The Fokker-Planck equation for the Ornstein-Uhlenbeck process reads

$$\frac{\partial}{\partial t}p = \frac{\partial}{\partial x}(kxp) + D\frac{\partial^2}{\partial x^2}p$$

- With $k > 0$ and $x \in (-\infty, \infty)$ show that the moment generating function is

  $$\phi(s,t) = Z(is,t) := \int_{-\infty}^{\infty} dx e^{isx} p(x,t) = \exp\left[-\frac{Ds^2}{2k}\left(1 - e^{-2kt}\right)\right]$$

  for initial condition $p(x, 0) = \delta(x)$.

- With $k < 0$, find the stationary distribution $p^*(x)$ on the interval $(-a, a)$ subject to periodic boundary conditions.

Aufgabe 7 (5 Punkte): *Kramers-Moyal expansion for the symmetric random walk*

Consider the master equation of the symmetric random walk

$$\dot{p}_n = p_{n+1} + p_{n-1} - 2p_n$$

For large n, p_n can be approximated as a continuous function of n, and we can approximate the master equation by performing a Taylor expansion of $p_{n\pm1}$ about n to second order. Solve the resulting Fokker-Planck equation for $p_n(t)$ and compare graphically with the solution of the full master equation (at both long and short times).

Vorlesung:
- Donnerstags 10:15 Uhr – 12:00 Uhr im EW 203.
- Freitags 10:15 Uhr – 12:00 Uhr im EW 203.

Übung:
- Montags 12:15 Uhr – 14:00 Uhr im EW 561

Scheinkriterien:
- Mindestens 50% der Übungspunkte.
- Regelmäßige und aktive Teilnahme in den Tutorien.
- Bearbeitung und Vorstellung eines Projektes (Projektvorstellung in der letzten Vorlesungswoche).

Literatur zur Lehrveranstaltung:

Siehe auch Semesterapparat in der Physikbibliothek.

- Crispin W. Gardiner, Handbook of stochastic method, Springer (2004)
- Nicolas G. van Kampen, Stochastic processes in physics and chemistry, North-Holland Publ. (2008)
- Ruslan L. Stratonovich, Topics in the Theory of Random Noise, Vols. I and II, Gordon and Breach (1963)
- Hannes Risken; Till Frank, The Fokker-Planck Equation, Methods of Solutions and Applications, Springer Berlin (1996)
- H. Haken, Quantenfeldtheorie des Festkörpers, Teubner (1973)
- H. Haug, S. W. Koch, Quantum Theory of the optical and electronic properties of semiconductors, World Scientific (2001)
- M. O. Scully, Quantum Optics, Cambridge University Press (1997)
- Scherz, Quantenmechanik, Teubner (2005)