Synchronization in Nonlinear Systems and Networks

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Wednesday 13:00-14:00

Lecture 4 - 23.11.2011

Chaos actually … is everywhere

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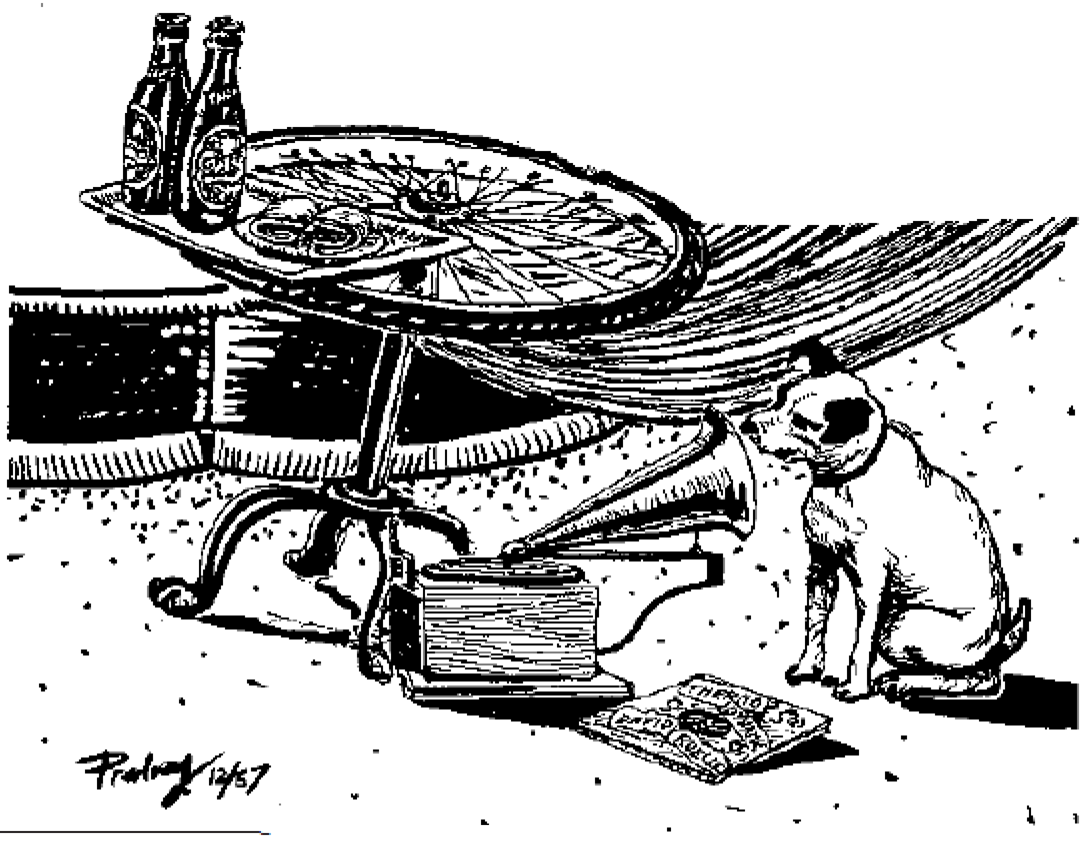
Chaos: Classical and Quantum

I: Deterministic Chaos

Predrag Cvitanović – Roberto Artuso – Ronnie Mainieri – Gregor Tanner – Gábor Vattay



Web Book

ChaosBook.org version13.3, Sep 23 2010
ChaosBook.org

printed September 23, 2010
comments to: predrag@nbi.dk

CHAOS = BUTTERFLY EFFECT

Henri Poincaré (1880)

" *It so happens that small differences in the initial state of the system can lead to very large differences in its final state.*

A small error in the former could then produce an enormous one in the latter. Prediction becomes impossible, and the system appears to behave randomly."

Ray Bradbury *"A Sound of Thunder "* (1952)

THE ESSENCE OF CHAOS

- processes ***deterministic***

  fully determined by initial state

- long-term behavior ***unpredictable***

  butterfly effect

PHYSICAL "DEFINITION" OF CHAOS

"To say that a certain system exhibits chaos means that the system obeys deterministic law of evolution but that the outcome is highly sensitive to small uncertainties in the specification of the initial state. In chaotic system any open ball of initial conditions, no matter how small, will in finite time spread over the extent of the entire asymptotically admissible phase space"

Predrag Cvitanovich . Appl.Chaos 1992

EXAMPLES OF CHAOTIC SYSTEMS

- the solar system (Poincare)
- the weather (Lorenz)
- turbulence in fluids
- population growth
- lots and lots of other systems…

"HOT" APPLICATIONS

- neuronal networks of the brain
- genetic networks

UNPREDICTIBILITY OF THE WEATHER

Edward Lorenz (1963)

Difficulties in predicting
the weather are not related
to the complexity of the
Earths' climate but to
CHAOS
in the climate equations!

Dynamical systems

Dynamical system: a system of one or more variables which evolve in time according to a given rule

Two types of dynamical systems:

- Differential equations: time is continuous (called flow)

$$\frac{dx}{dt} = f(x), \qquad t \in R^{N}$$

- Difference equations (iterated maps): time is discrete (called cascade)

$$x_{n+1} = f(x_n), \qquad n = 0, 1, 2, \ldots$$

Attractor

is a closed set with the following properties:

1. A is an *invariant set*: any trajectory $\mathbf{x}(t)$ that starts in A stays in A for all time.
2. A *attracts an open set of initial conditions*: there is an open set U containing A such that if $\mathbf{x}(0) \in U$, then the distance from $\mathbf{x}(t)$ to A tends to zero as $t \to \infty$. This means that A attracts all trajectories that start sufficiently close to it. The largest such U is called the *basin of attraction* of A.
3. A is *minimal*: there is no proper subset of A that satisfies conditions 1 and 2.

S. Strogatz

Different types of attractors . Strange attractor

An attractor *A* is a set in phase space, towards which a dynamical system evolves over time. This limiting set *A* can be:

1) point (equilibrium)
2) curve (periodic orbit)
3) torus (quasiperiodic orbit)

Up to the beginning of 60th people believed that nothing more complicated is possible in deterministic systems. But:

4) fractal set (*strange attractor = CHAOS*)

"…I'm strangely attracted to you"
Cole Porter (1953)

MATHEMATICAL DEFINITION OF CHAOS

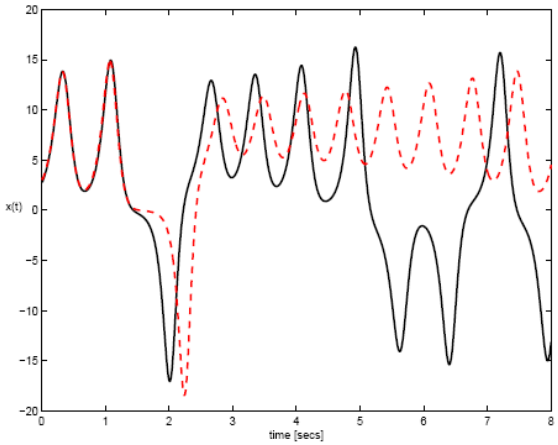
Chaos is *aperiodic* long-term behavior in a *deterministic* system that exhibits sensitive dependence on initial conditions caused by *strange attractor* A:

The mappins $f : A \to A$ is said to be chaotic on A if:

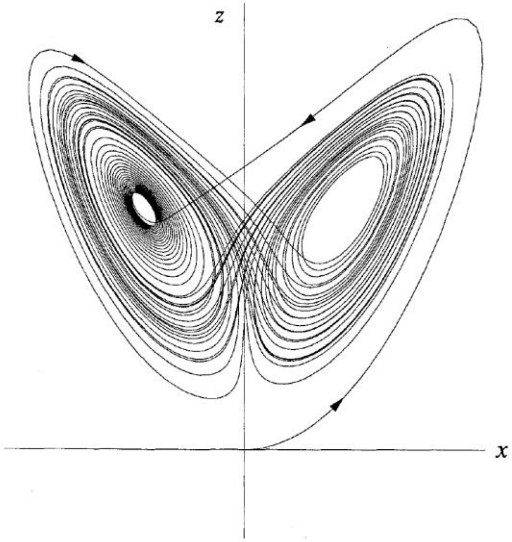
1. f has sensitive dependence on initial conditions
2. f is topologically transitive
3. periodic points are dense in A

LORENTZ ATTRACTOR (1963)

butterfly effect



a trajectory in the phase space



The Lorenz attractor is generated by the system of 3 differential equations

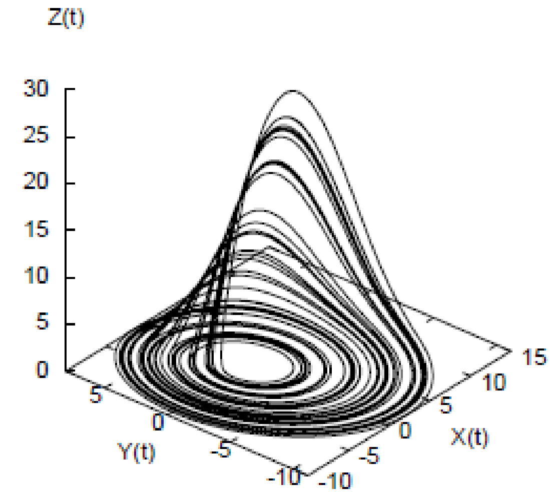
$$dx/dt = -10x + 10y$$

$$dy/dt = 28x - y - xz$$

$$dz/dt = -8/3z + xy.$$

ROSSLER ATTRACTOR (1976)

$$\dot{x} = -y - z$$
$$\dot{y} = x + ay$$
$$\dot{z} = b + z(x - c), \qquad a = b = 0.2, \quad c = 5.7$$



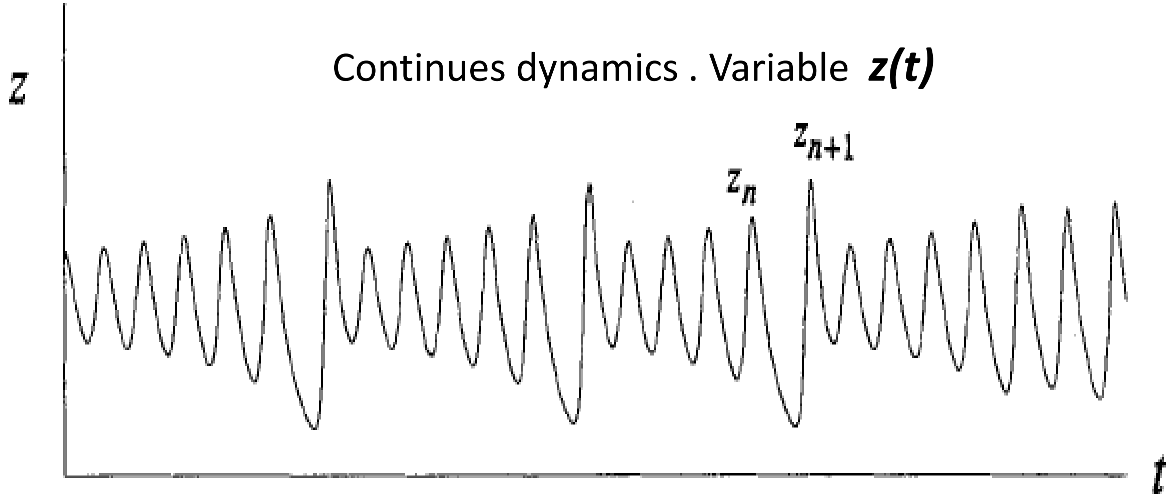
A trajectory of the Rossler system, t=250

To see what solutions looks like in general, we need to perform numerical integration.
One can observe that trajectories looks like behave chaotically and
converge to a *strange attractor*.
But, there exists NO any mathematical proof that such attractor is asymptotically *aperiodic.*
It might well be that what we see is but a long transient on a way to an attractive periodic orbit

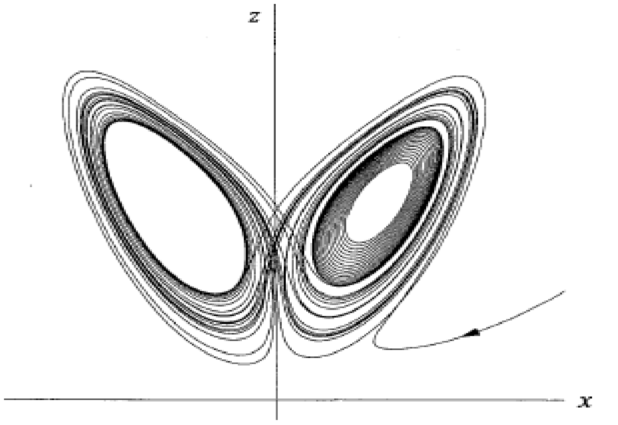
What have we learned from the above two exemplary 3-dimensional flows? If a flow is locally unstable but globally bounded, any open ball of initial points will be stretched out and then folded back. If the equilibria are hyperbolic, the trajectories will be attracted along some eigen-directions and ejected along others.

Reducing to discrete dynamics. Lorenz map

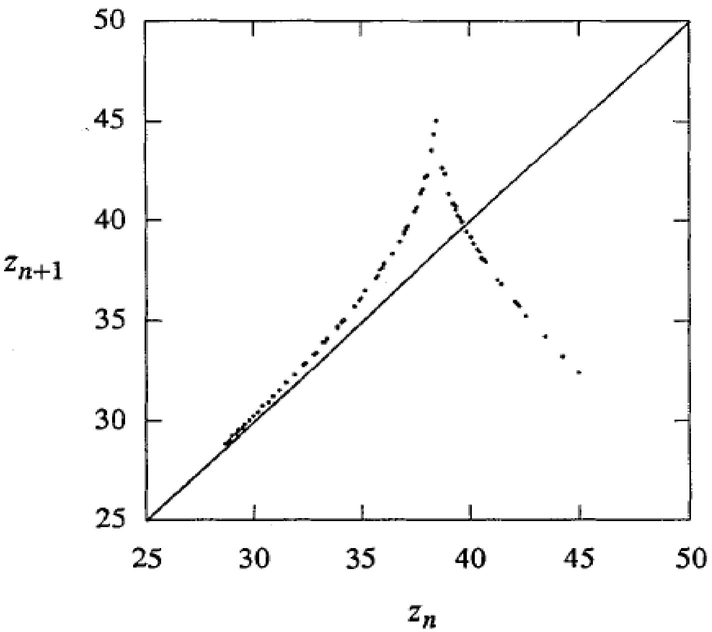
Continues dynamics . Variable ***z(t)***



Lorenz attractor



Lorenz one-dimensional map



$$z_{n+1} = f(z_n)$$

Poincare section and Poincare return map

Rossler one-dimensional map

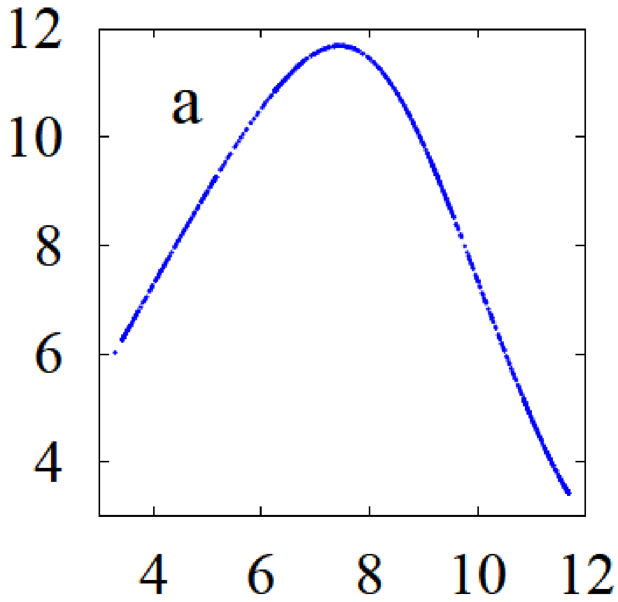


Figure 3.6: Return maps for the $R_n \to R_{n+1}$ radial distance Poincaré sections of figure 3.5. (R. Paškauskas)

Rossler attractor