

## English summary:

### 2.2 SNIPER model

Saddle-node infinite period bifurcation

$$\begin{cases} \dot{x} = x(1-x^2-y^2) + y(x+b) \\ \dot{y} = y(1-x^2-y^2) - x(x+b) \end{cases} \Leftrightarrow \begin{cases} \dot{r} = r(1-r^2) \\ \dot{\varphi} = b - r \cos \varphi \end{cases} \left| \begin{array}{l} \text{LC for } |b| > 1 \\ \text{with period } T = \frac{2\pi}{\sqrt{b^2-1}} \end{array} \right.$$

Fixed points and their stability ( $\dot{x}=0, \dot{y}=0$ , linear stability analysis)

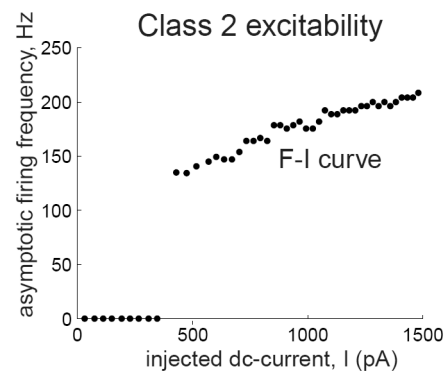
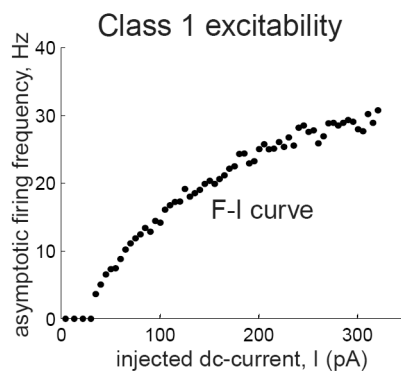
$$(x_A^*, y_A^*) = (0, 0) \Rightarrow \text{Eigenvalues } \lambda_{1,2} = 1 \pm ib \Rightarrow \text{unstable focus}$$

$$(x_B^*, y_B^*) = (b, +\sqrt{1-b^2}) \Rightarrow \lambda_1 = -2, \lambda_2 = +\sqrt{1-b^2} \Rightarrow \text{saddle node}$$

$$(x_C^*, y_C^*) = (b, -\sqrt{1-b^2}) \Rightarrow \lambda_1 = -2, \lambda_2 = -\sqrt{1-b^2} \Rightarrow \text{stable node}$$

Limit cycle for  $|b| > 1$ : period diverges for  $|b| \rightarrow 1$

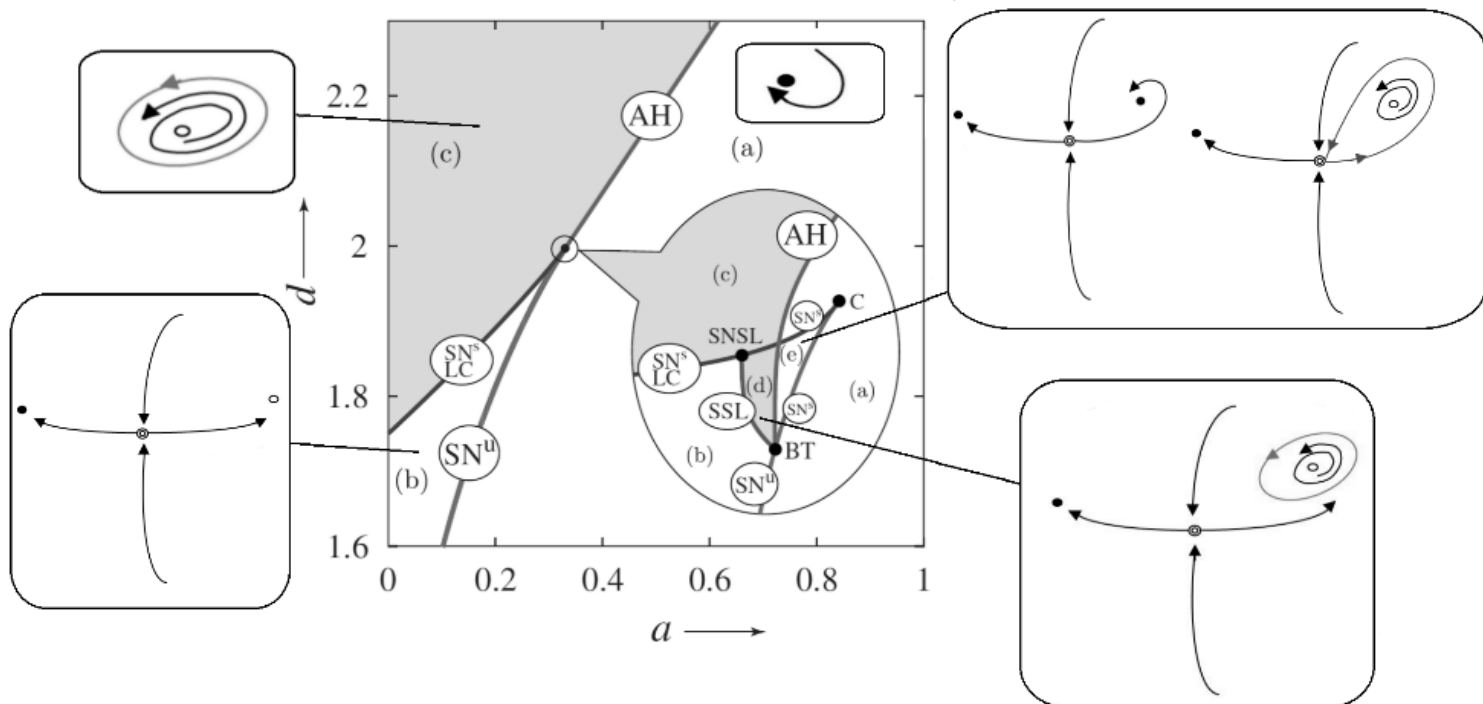
$\Rightarrow$  excitability type I  
(frequency = 0 at bif.)



### 2.3 Hindmarsh-Rose model (2D)

$$\begin{cases} \dot{x} = c \left( x - \frac{x^3}{3} - y + z \right) \\ \dot{y} = \frac{1}{c} (x^2 + dx - by + a) \end{cases} \left. \begin{array}{l} \text{FPs} \Leftrightarrow \text{intersection of nullclines} \\ \Rightarrow 1, 2, \text{ or } 3 \text{ FPs (roots of cubic polynomial)} \end{array} \right\}$$

LSA:  $J = \begin{pmatrix} c - c(x^*)^2 & -c \\ \frac{1}{c}(2x^* + d) & -\frac{b}{c} \end{pmatrix}$  depends only on  $x^*$  coordinate



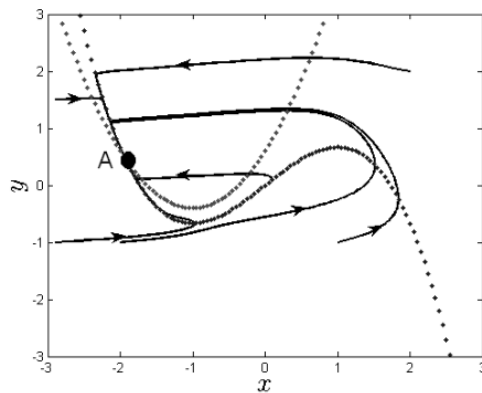
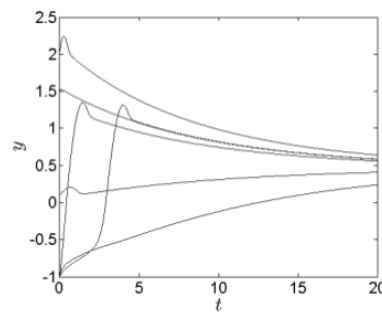
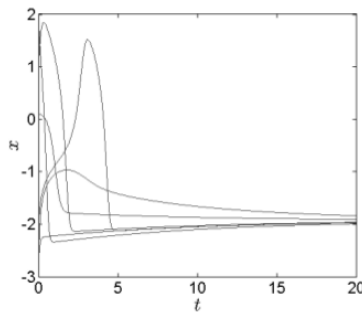
### 2.3 Hindmarsh-Rose-Modell (Fortsetzung)

Szenario ( $b=1, c=3, z=0$ ):

(a)  $a=0.6, d=1.7$

Stabiler Knoten

$$\lambda_{1,2} < 0$$



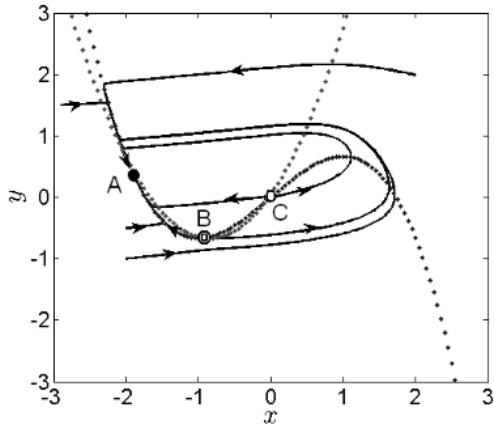
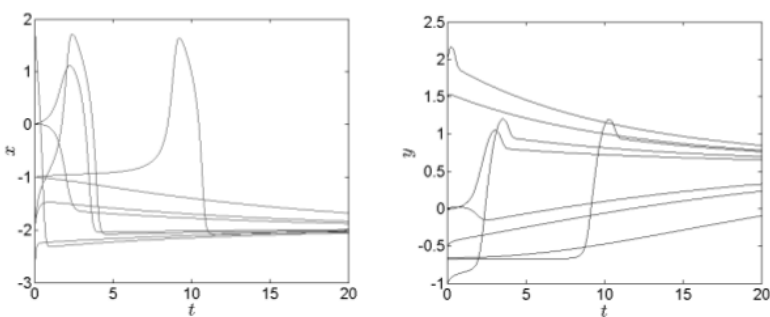
(b)  $a = 0.05, d = 1.7$

3 Fixpunkte:

A: stabiler Knoten  $\lambda_{1,2} < 0$

B: instabiler Knoten  $\lambda_{1,2} > 0$

C: Sattelpunkt  $\lambda_1 < 0, \lambda_2 > 0$

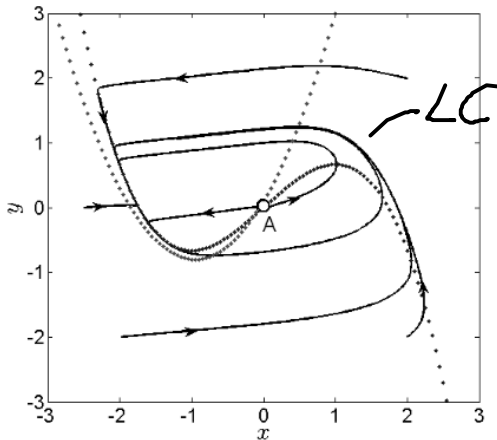
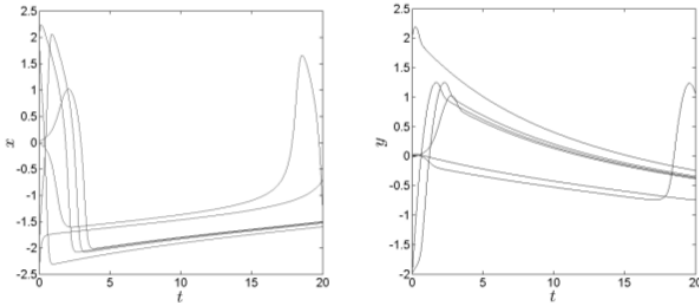


(c)  $a = 0.1, d = 1.9$

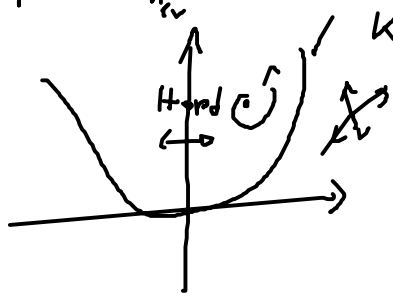
1 Fixpunkt

instabiler Knoten

$\lambda_{1,2} > 0$



Übergang  
instabil Fokus  $\rightarrow$  instabiler Knoten



Hindmarsh-Rose-Modell mit 3 Variablen

$$\dot{x} = y - ax^3 + bx^2 - z + I \quad \text{kubisch}$$

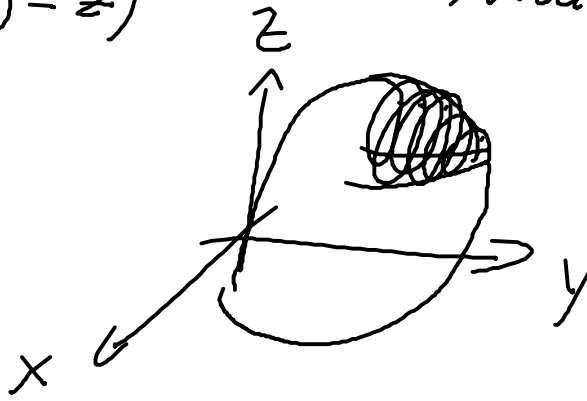
$$\dot{y} = c - dx^2 - y$$

quadratisch

$$\dot{z} = \epsilon (\sigma (x - x_0) - z)$$

linear

$\Rightarrow$  Bursting



### 3 Physiologische Modelle

#### ► Physiological models:

- e.g. Hodgkin-Huxley equations
- Many physiological details and processes
- Detailed description of single cell
- Many equations, many parameters
- Applicable to ensembles of many oscillators?
- Feasible for bifurcation analysis?

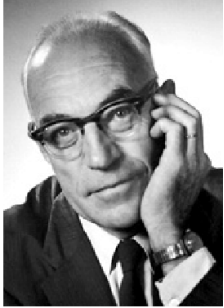
3.1 Hodgkin-Huxley-Modell

3.2 Morris-Lecar-Modell

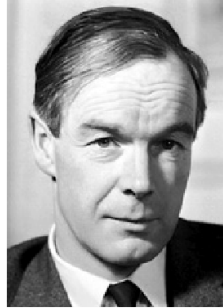
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3.1 Hodgkin-Huxley-Modell

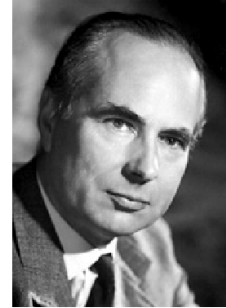
"for their discoveries concerning the ionic mechanisms involved in excitation and inhibition in the [...] nerve cell membrane"



Sir John Carew Eccles



Alan Lloyd Hodgkin



Andrew Fielding Huxley

\* 5.02.1914  
† 20.12.1988

\* 22.11.1917  
† 30.05.2012

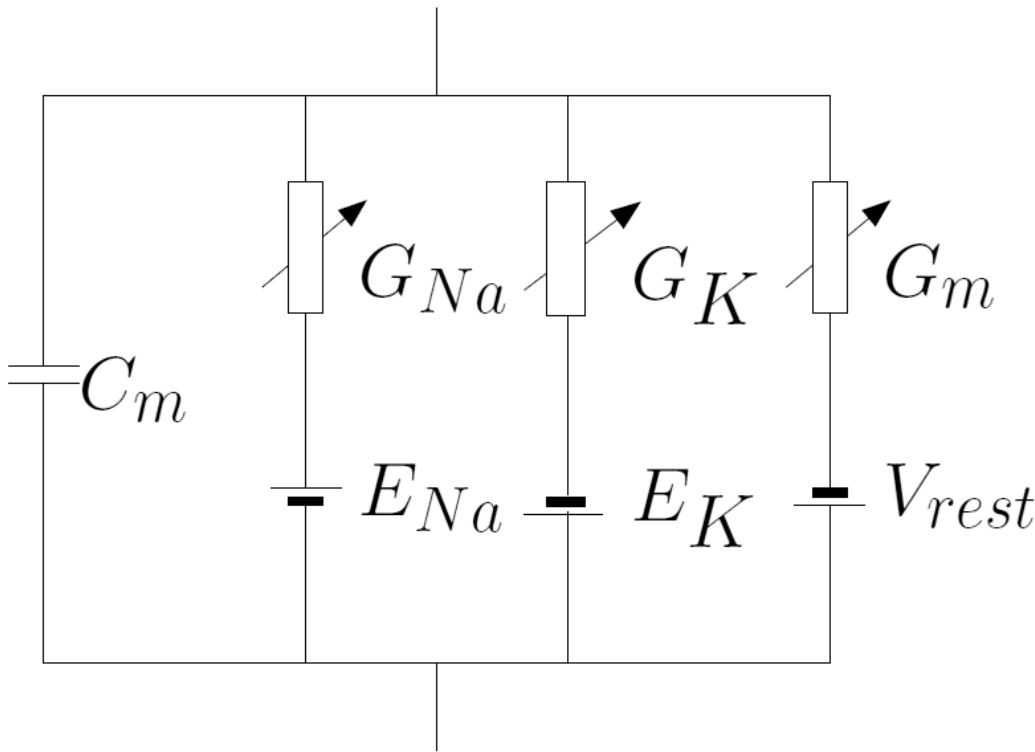
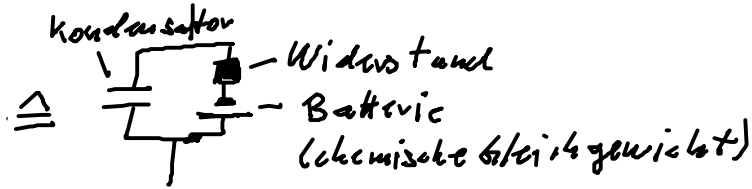
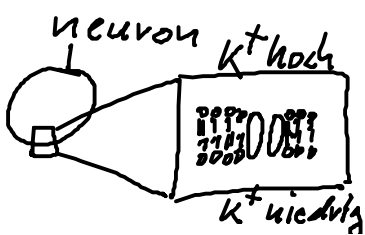
**J. Physiol. (1952) 117, 500-544**

## **A QUANTITATIVE DESCRIPTION OF MEMBRANE CURRENT AND ITS APPLICATION TO CONDUCTION AND EXCITATION IN NERVE**

**BY A. L. HODGKIN AND A. F. HUXLEY**

*From the Physiological Laboratory, University of Cambridge*

# Conductance-based model



Kirchhoff-Gesetz ( $\sum_{k=1}^n I_k = 0$  innerhalb einer Masche)  
 an einem Knoten

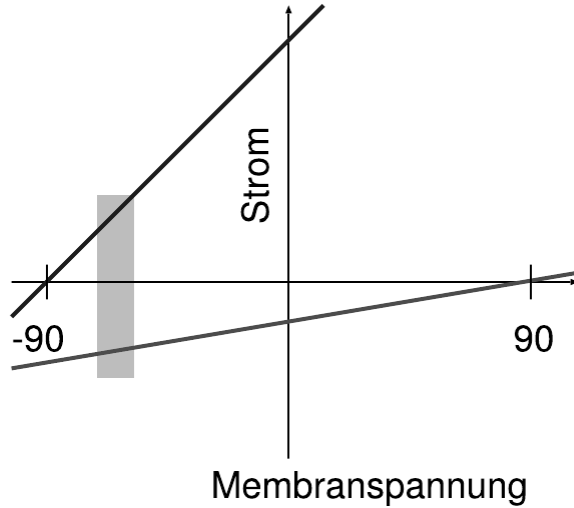
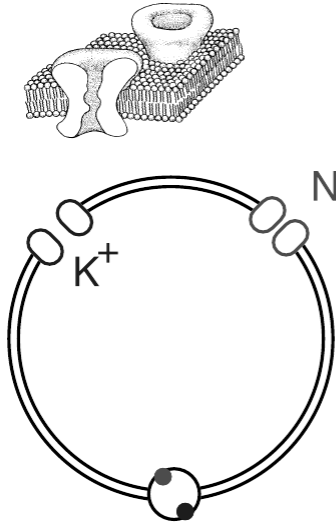
$$\underset{\substack{\uparrow \\ \text{Membran}}}{I_m} = - \underbrace{(I_{Na} + I_K + I_{leak})}_{I_{ionic}}$$

Spannung am Kondensator (Membran):  $Q = C U$

$$\dot{Q} = C \dot{U} \Rightarrow C \dot{U} = I_m = - I_{ionic}$$

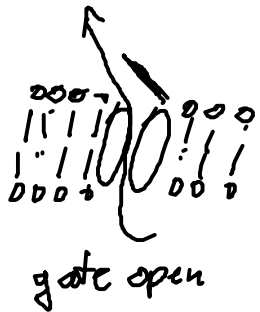
Ohmsches Gesetz:  $U = R I \Rightarrow \underset{g = \frac{1}{R}}{g} (V - E_{ion}) = I_{ion}$

$E_{ion}$  (Batterie) ergibt sich aus Nernst-Potenzial bei unterschiedlichen Ionenkonzentrationen:  $E_{ion} \sim \ln \frac{[Ion]_{in}}{[Ion]_{out}}$   
 Ursachen: Balance zwischen Diffusion und elektrostatischen Drift

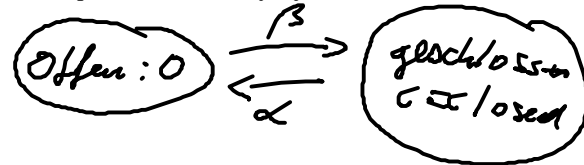


Achtung: Leitfähigkeit  $g_{ion}$  ist nicht linear / hängt von  $V$

$\Rightarrow g_{ion} = \bar{g}$  "gating variable"  
 ↑ max. Leitfähigkeit      ↑ spannungsabhängig



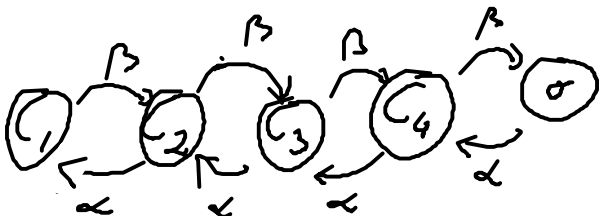
$g(V) = \bar{g} p(V)$   
 ↑ Wahrscheinlichkeit für "offen"



$\dot{p} = -\beta p + \alpha (1-p)$

angewendet auf K und Na:

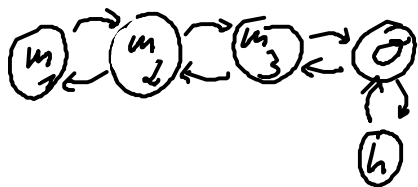
$I_K = \bar{g}_K n^4 (V - E_K)$  mit  $n = \alpha_n(V) (1-n) - \beta_n(V) n$



$\alpha_n(V) = 0.01 \frac{10 - V}{e^{(10-V)/10} - 1}$   
 $\beta_n(V) = 0.125 e^{-V/80}$

$$I_{Na} = \bar{g}_{Na} m^3 h (V - E_{Na}) \text{ mit}$$

Zunabhängige Karäle



$$\dot{m} = \alpha_m(V) (1-m) - \beta_m(V) m$$

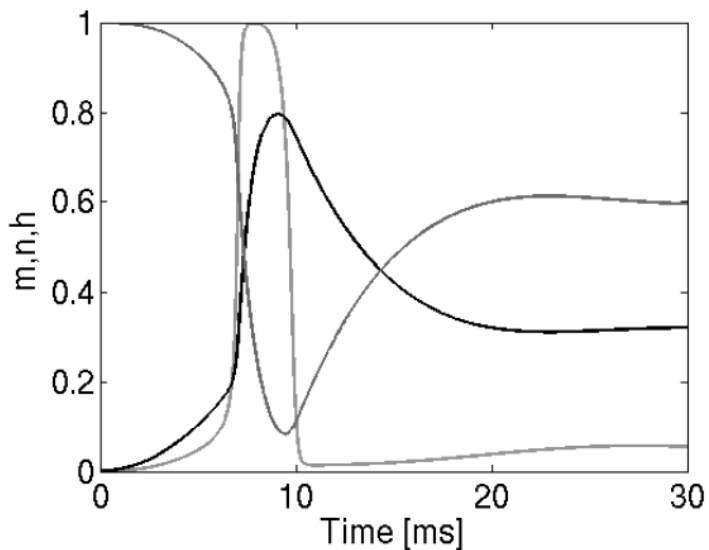
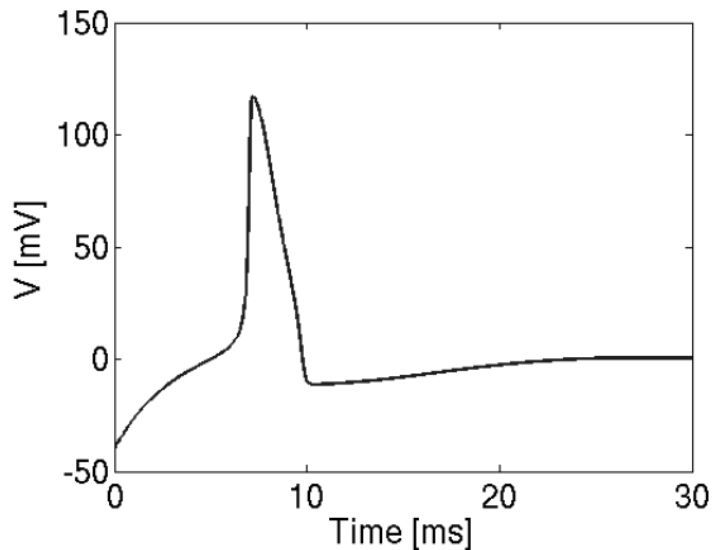
$$\dot{h} = \alpha_h(V) (1-h) - \beta_h(V) h$$

$$\alpha_m(V) = 0.1 \frac{25 - V}{e^{(25-V)/10} - 1}$$

$$\beta_m(V) = 4e^{-V/18}$$

$$\alpha_h(V) = 0.07e^{-V/20}$$

$$\beta_h(V) = \frac{1}{e^{(30-V)/10} + 1}$$



Left panel: Membrane voltage. Right panel: Activation potential, for sodium activation  $m$  (green) and inactivation  $h$  (pink) and potassium activation  $n$  (black).

Gleichung für Membranpotential:

$$C_m \dot{V} = -\bar{g}_K n^4 (V - E_K) - \bar{g}_{Na} m^3 h (V - E_{Na}) - \bar{g}_{Leak} (V - E_{Leak})$$

Fortsetzung folgt ...