4 Interplay of noise and delay

4.1 Fundamentals

Langevin equation: \( m \dot{x} = -\eta x + \xi(t) \) (e.g., Brownian motion)

Stochastic differential eq.: \( dx_t = \Theta(\mu - x_t) dt + \sigma dW_t \)

\( W_t \): random variable of a Wiener process
\( W_0 = 0 \), increment: \( W_t - W_s \sim N(0, t-s) \) normal distribution with mean 0, variance \( t-s \)

Gaussian white noise:
\( \langle \xi(t) \rangle = 0 \) (no net force)
\( \langle \xi(t) \xi(s) \rangle = \delta(t-s) \) (uncorrelated, no memory)

\( \tau^2(s) = \langle (x(t) - \langle x \rangle) (x(t-s) - \langle x \rangle) \rangle \)

Power spectral density:
\( S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \langle x(t) x(t-s) \rangle e^{i\omega s} \, ds \) (for \( \langle x \rangle = 0 \))

\( \Rightarrow \) Wiener-Khintchine theorem (\( S(\omega); \) Fourier transform of \( \tau^2(s) \))

4.2 Coherence resonance

- constructive influence of noise
- noise-induced oscillations most regular for finite/optimal
Continuos control of chaos by self-controlling feedback

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Refuting the Odd-Number Limitation of Time-Delayed Feedback Control

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\[ \hat{z}(t) = [\lambda + i + (1 + i\gamma)|\hat{z}(t)|^2]\hat{z}(t) + b[\hat{z}(t - \tau) - \hat{z}(t)] \]
\( z \in \mathbb{C} \), Kontrollstärke \( b = b_0 + ib_1 \in \mathbb{C} \)

Geländefunktionsparameter \( R \in [-1, 1] \)

Extended Pyragas control: \( \mathcal{F}(t) = b \sum_{n=0}^{\infty} R^n \left[ z(t-n) - \lambda z(t-n) \right] \)

\( \text{Exp. } \rightarrow \text{Applet} \quad \lambda = -0.005 \Rightarrow |z_{c0}| = \sqrt{-\lambda} \approx 0.071 \)

**FIG. 3 (color online).** Domain of control in the plane of the complex feedback gain \( b = b_0 e^{i\beta} \) for three different values of the bifurcation parameter \( \lambda \). The black solid curves indicate the boundary of stability in the limit \( \lambda \not\rightarrow 0 \); see (18) and (19). The color-shading shows the magnitude of the largest (negative) real part of the Floquet exponents of the periodic orbit (\( \gamma = -10, \tau = \frac{2\pi}{1-\gamma\lambda} \)).

Anwendung auf neuronale Dynamik.
Delay control of coherence resonance in type-I excitable dynamics

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\[
\begin{align*}
\dot{x} &= x (1 - x^2 - y^2) + y (x - b) + D \xi + K (x_\tau - x) \\
\dot{y} &= y (1 - x^2 - y^2) - x (x - b) + D \xi + K (y_\tau - y).
\end{align*}
\]

(1a) (1b)

Here \(x\) and \(y\) are the variables at time \(t\), while \(x_\tau\) and \(y_\tau\) denote the respective variables at a delayed time \(t - \tau\). The bifurcation parameter \(b \in \mathbb{R}\) is a real constant. \(K\) denotes the control strength and \(\tau\) is the delay time. Random input \(\xi\) is realized as Gaussian white noise with mean \(\langle \xi(t) \rangle = 0\), variance \(\langle \xi(t) \xi(t') \rangle = \delta(t - t')\), and noise intensity \(D\). In polar coordinates the system equations for \(D = 0\) and \(K = 0\) are given by [22]:

\[
\begin{align*}
\dot{r} &= r (1 - r^2) \\
\dot{\phi} &= b - r \cos \phi.
\end{align*}
\]

(2a) (2b)

Ziel: Kontrolle (Verstärkung der Regularität) rausch induzierten Oscillationen.
Correlation time in dependence on the noise intensity $D$ for different time delays $\tau$. The solid (green) curve corresponds to the uncontrolled system ($\tau = 0$). The dashed (red), dash-dotted (blue), and dotted (black) curves refer to values of $\tau = 2, 5, \text{and} 9$, respectively. Other parameters: $b = 0.95$ and $K = 0.25$.

Correlation time $t_{cor}$ in dependence on the time delay $\tau$ for two values of the noise intensity $D$. The dashed (red) curve corresponds to $D_{subopt} = 0.25$ and the solid (blue) curve refers to $D_{opt} = 0.15$. Other parameters: $b = 0.95$ and $K = 0.25$. 
Correlation time $t_{cor}$ in dependence on the control strength $K$ for two values of the noise intensity $D$ and two values of the delay time $\tau$. The gray (red) and black (blue) curves depict the cases of $D_{subopt}$ and $D_{opt}$, respectively. The solid and dashed lines correspond to $\tau = 2$ and $\tau = 9$, respectively. Other parameter: $b = 0.95$. 

5. Dynamische gekoppelter Elemente