

English summary

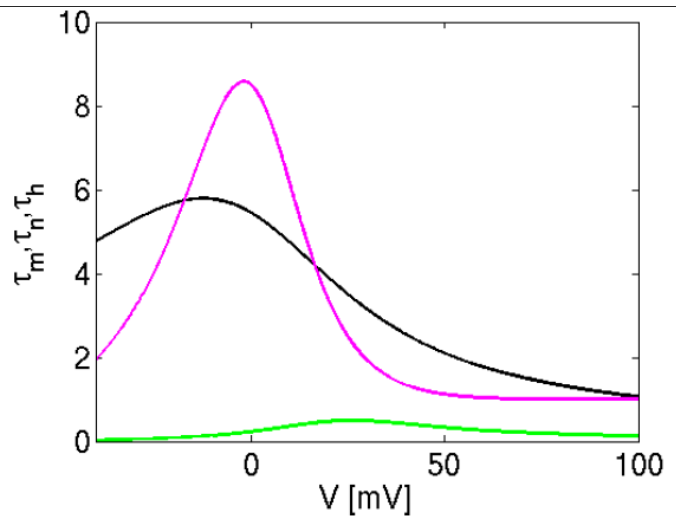
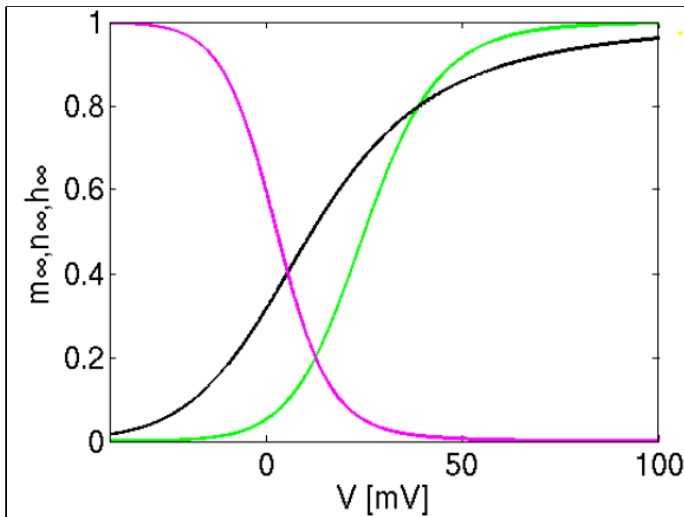
3.1 Hodgkin-Huxley model

membrane potential

$$C_m \dot{V} = I - g_K n^4 (V - E_K) - g_{Na} m^3 h (V - E_{Na}) - g_L (V - E_L)$$

gating variables

$$\begin{cases} \tau_n(V) \dot{n} = -n + n_\infty(V) & n \text{ activation} \\ \tau_m(V) \dot{m} = -m + m_\infty(V) & m \text{ activation} \\ \tau_h(V) \dot{h} = -h + h_\infty(V) & h \text{ inactivation} \end{cases}$$



Left panel: Steady state activation and inactivation functions. Right panel: Voltage dependent time constants, for sodium activation m (green) and inactivation h (pink) and potassium activation n (black).

3.2 Morris-Lecar model

$$C_m \dot{V} = I - g_K w(V) (V - E_K) - \underbrace{g_{Ca} m_\infty(V) (V - E_{Ca})}_{\text{fast Ca dynamics}} - g_L (V - E_L)$$

$$\tau_w(V) \dot{w} = -w + w_\infty(V)$$

and the following functions:

$$m_\infty(V) = \frac{1}{2} \left[1 + \tanh \left(\frac{V - V_2}{V_2} \right) \right]$$

$$w_\infty(V) = \frac{1}{2} \left[1 + \tanh \left(\frac{V - V_3}{V_4} \right) \right]$$

$$\tau_w(V) = \frac{1}{\cosh \left(\frac{V - V_3}{2V_4} \right)}$$

\Rightarrow homoclinic bifurcation (among others: Hopf, SNIPER)

4. Wechselspiel von Rauschen und Zeitverzögerung

bisher: deterministische dynamische Systeme

jetzt: stochastische dynamische Systeme

4.1 Grundbegriffe

4.2 Kohärenzresonanz

4.3 zeitverzögerte Rückkopplung

(gekoppelte Systeme \rightarrow Kap 5)

4.1 Grundbegriffe

stochastischer Prozess: Zeitentwicklung einer Zufallsvariable

Langevin-Gleichung: fluktuierende stochastische Kraft $f(t)$
(Rauschen, noise)

z.B. Brownsche Bewegung (1827)

$$m\ddot{x} = -\eta\dot{x} + f(t) \quad (\text{Bewegung eines Teilchens})$$

$\underbrace{\hspace{2cm}}_{\text{Reibung}} \quad \underbrace{\hspace{2cm}}_{\text{Rauschen (Kraft durch zufällige Stöße)}}$

stochastische Dgl: $dx_t = \theta(\mu - x_t) dt + \sigma dW_t$ Rauschen

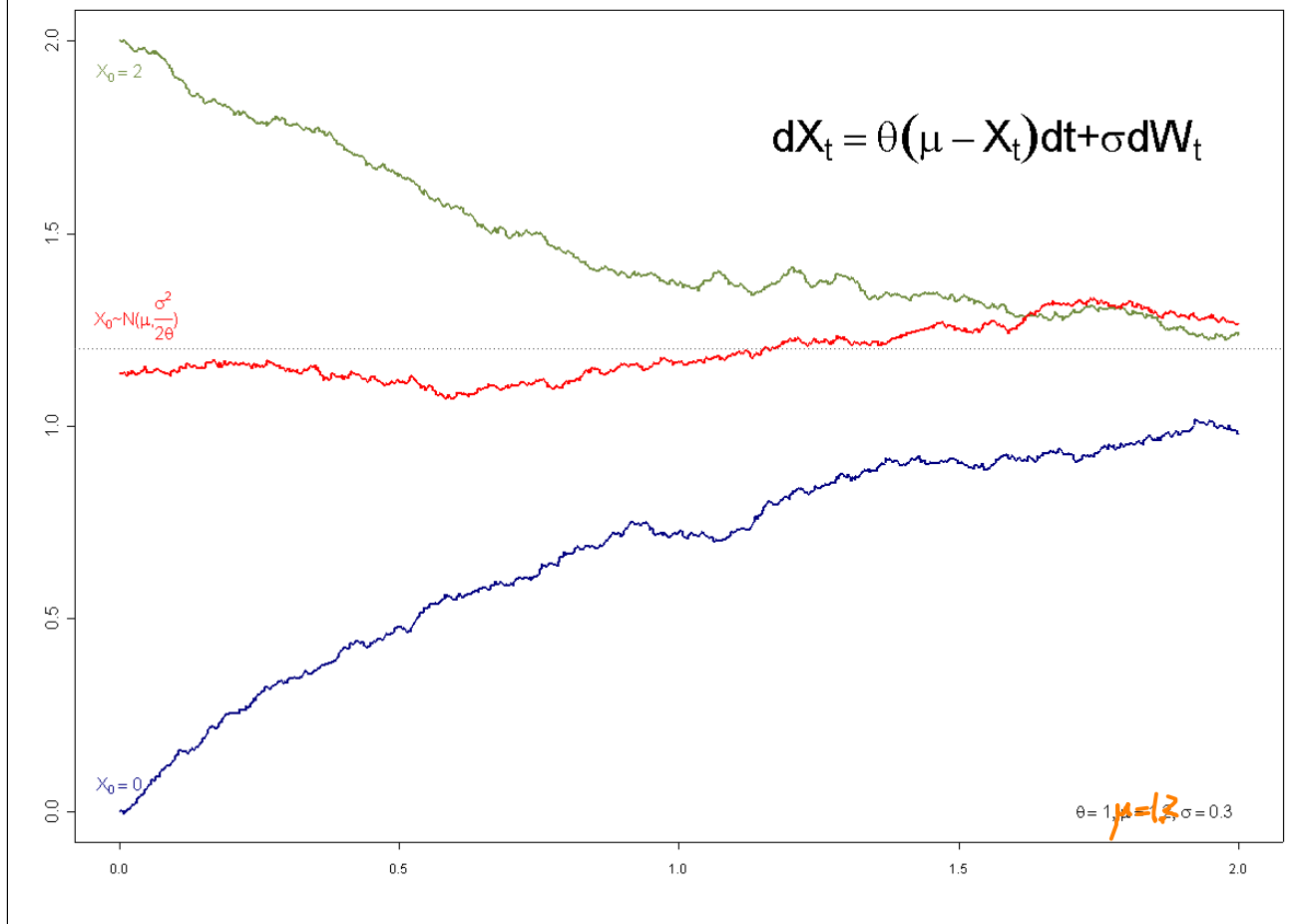
\uparrow \uparrow \uparrow \leftarrow \uparrow

Geschwindigkeit μ langfristige Mittelwert σ^2 Volatilität

Zeit θ Rückkehrzeit

Ornstein-Uhlenbeck-Prozess

Ornstein - Uhlenbeck



W_t : Zufallsvariable eines Wiener-Prozesses

$\Rightarrow W_0 = 0$

$W_t - W_s \sim N(0, t-s)$ Normalverteilung mit Mittelwert 0 und Varianz $t-s$

(Zentraler Grenzwertsatz: unkorrelierte Zufallsvariablen gehören einer Gauß-Verteilung)

Gaußsches weißes Rauschen: $\langle \xi(t) \rangle = 0$ (im Mittel keine gerichtete Kraft)

$\langle \xi(t) \xi(t') \rangle = \delta(t-t')$ (unkorreliert, keine Gedächtniseffekte)

Autokorrelationsfunktion (Vergleich mit sich selbst)

$$\Psi(s) = \left\langle \underbrace{(x(t) - \langle x \rangle)}_{\text{um } s \text{ verschoben}} \underbrace{(x(t+s) - \langle x \rangle)}_{\text{Zeitschieb}} \right\rangle_{t \in \mathbb{R}}$$

t ∈ ℝ Mittelung über Zeit

$$\bar{\Psi}(s) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T dt \, x(t) x(t+s) \quad (\text{wenn } \langle x \rangle = 0)$$

endliches Zeitintervall

Fourier-Transformation: $\hat{X}(\omega, T) = \frac{1}{2T} \int_{-T}^T dt \, e^{i\omega t} x(t)$

Spektrale Leistungsdichte (power spectral density): $S(\omega) = \lim_{T \rightarrow \infty} \frac{T}{2T} |\hat{X}(\omega, T)|^2$

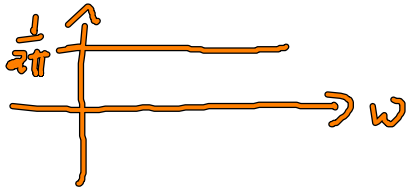
$$S(\omega) = \frac{1}{2T} \int_{-\infty}^{\infty} \langle x(t) x(t+s) \rangle e^{i\omega s} ds \quad (\text{wenn } \langle x \rangle = 0)$$

S ist die Fourier-Transformierte von $\bar{\Psi}(s)$ (Wiener-Khinchin-Theorem)

Bsp.: Gauß'sches weißes Rauschen $\xi(t)$:

$$S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \langle \xi(t) \xi(t+s) \rangle e^{i\omega s} ds$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \delta(s) e^{i\omega s} ds = \frac{1}{2\pi} = \text{const.}$$



alle Frequenzen gleichermaßen vertreten
 \Rightarrow "weiß"

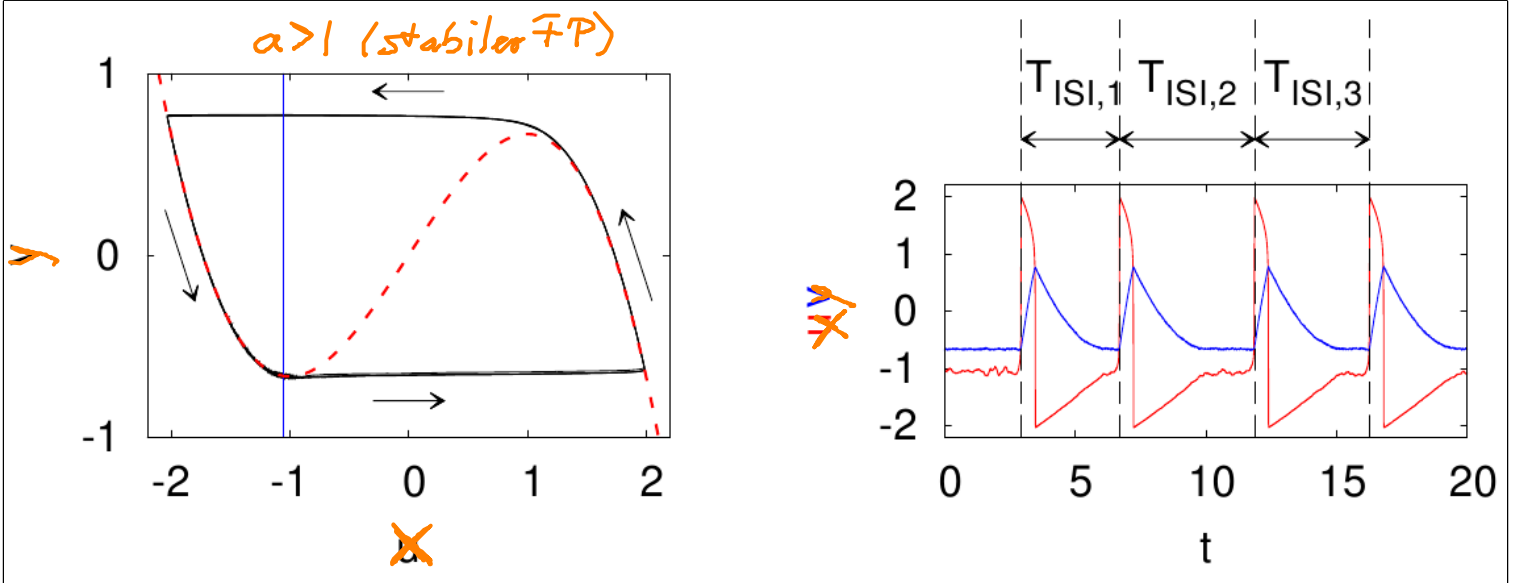
4.2 Kohärenzresonanz

- konstruktiver Einfluss von Rauschen
 \Rightarrow Regularität verschiedener Oszillationen am größten für eine endliche Rauschstärke

Bsp.: $\dot{x} = x - \frac{x^3}{3} - y$

$$\dot{y} = x + a + D \xi(t)$$

fast periodisches Verhalten



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Stochastic Resonance without External Periodic Force

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Coherence Resonance in a Noise-Driven Excitable System

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We study the dynamics of the excitable Fitz Hugh–Nagumo system under external noisy driving. Noise activates the system producing a sequence of pulses. The coherence of these noise-induced oscillations is shown to be maximal for a certain noise amplitude. This new effect of coherence resonance is explained by different noise dependencies of the activation and the excursion times. A simple one-dimensional model based on the Langevin dynamics is proposed for the quantitative description of this phenomenon. [S0031-9007(97)02349-1]

Maß für Regularität: Korrelationszeit: $t_{cor} = \frac{1}{\Psi(0)} \int_0^{\infty} |\Psi(s)| ds$

normierte Fluktuationen der interspikeintervalle

$$R = \frac{\sqrt{\langle T_{ISI}^2 \rangle - \langle T_{ISI} \rangle^2}}{\langle T_{ISI} \rangle}$$

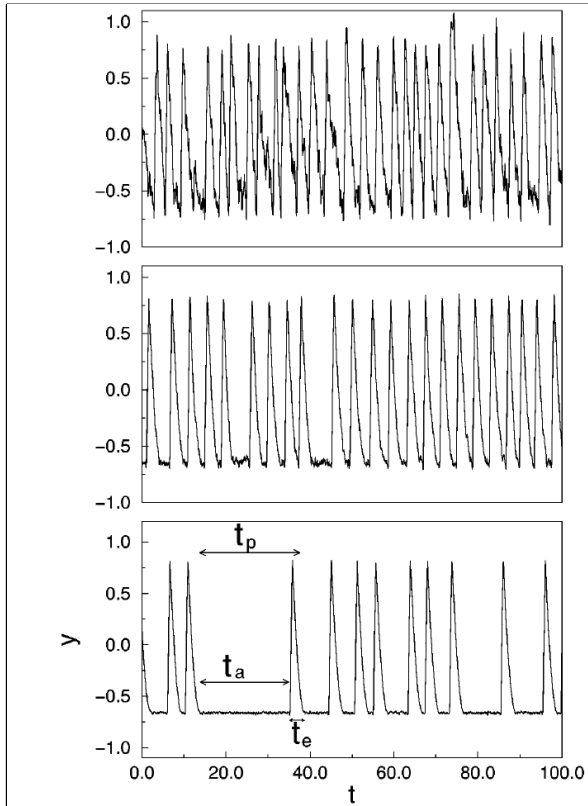


FIG. 1. The dynamics of the Fitz Hugh–Nagumo system [Eqs. (1), (2)] for $a = 1.05$, $\varepsilon = 0.01$, and different noise amplitudes: From bottom to top $D = 0.02$, $D = 0.07$, and $D = 0.25$. The mean durations of pulses are 7, 4, and 3.5, respectively. The activation and the excursion times for one pulse are depicted.

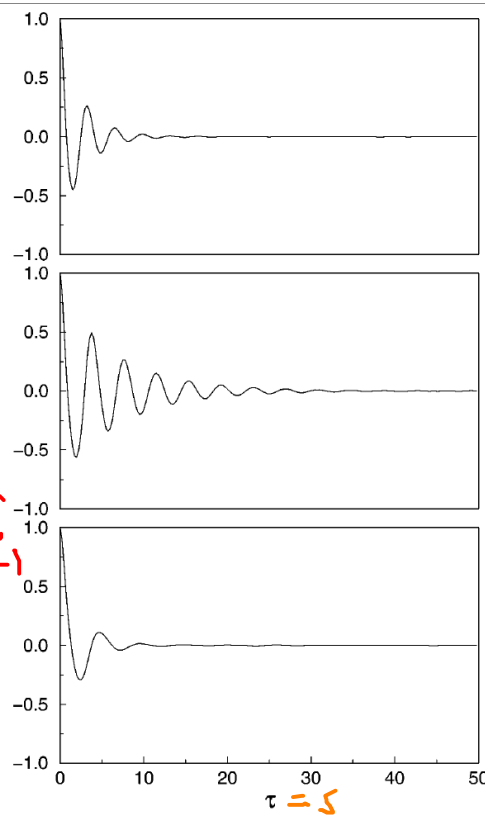


FIG. 2. The autocorrelation function of the regimes presented in Fig. 1.

Physically, the appearance of coherence resonance is

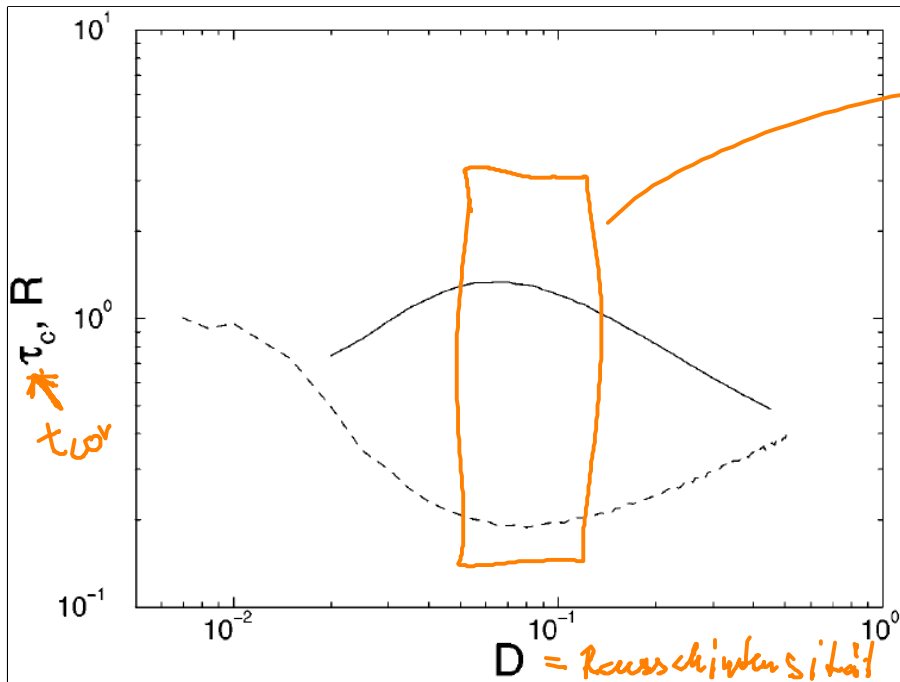
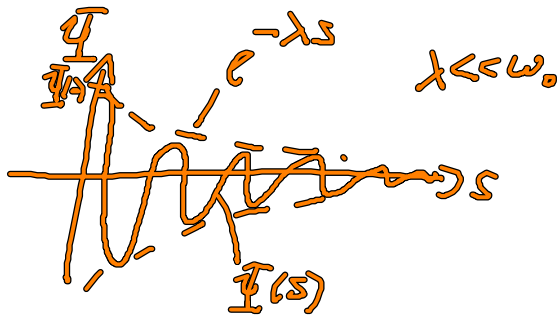


FIG. 3. Correlation time τ_c (solid line) and the noise-to-signal ratio R [Eq. (5), dashed line] vs noise amplitude for the Fitz Hugh–Nagumo system with $a = 1.05$, $\varepsilon = 0.01$.

• Korrelationszeit maximal
 • Fluktuation von T_{ISI} minimal
 ⇒ optimale Rauschstärke

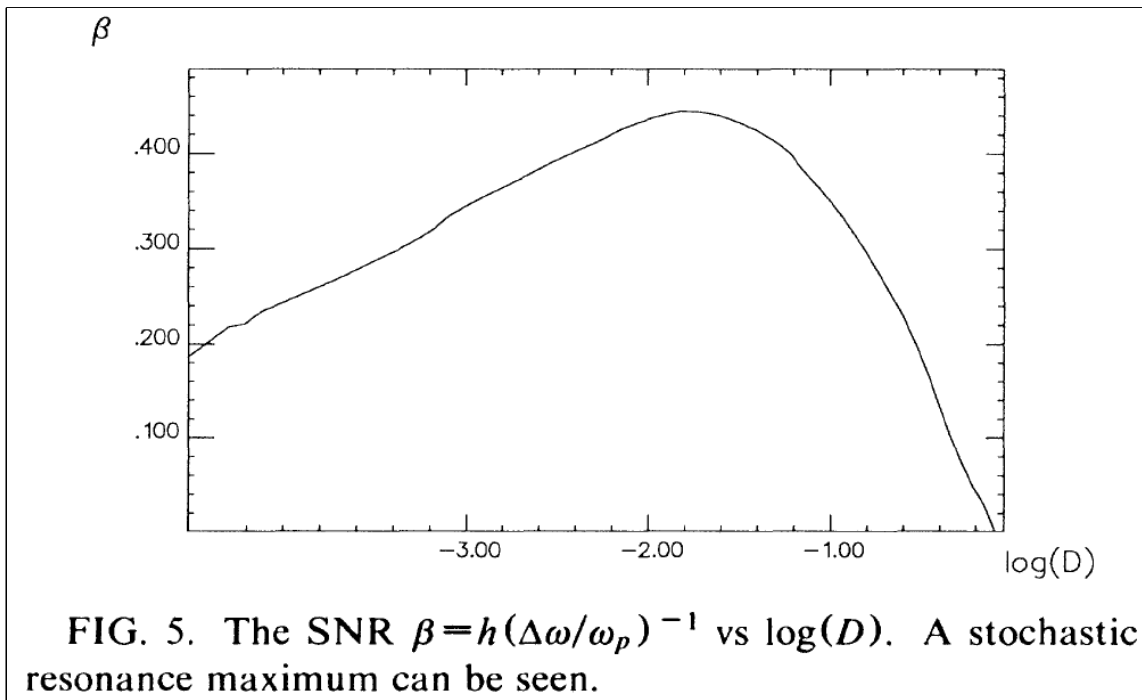
Lösung linearer stochastischer Prozess: $\dot{x} = -(A + i\omega_0)x + \xi(t)$

$$\Rightarrow \bar{\psi}(s) = \bar{\psi}(0) e^{-\lambda s} \cos(\omega_0 s)$$



$$\tau_{\text{cor}} = \int_0^{\infty} e^{-\lambda s} |\cos \omega_0 s| ds \approx \frac{2}{\pi \lambda}$$

$$\Rightarrow \bar{\psi}(s) = \bar{\psi}(0) e^{-\frac{2}{\pi} \frac{s}{\tau_{\text{cor}}}} \cos(\omega_0 s)$$



Signal-Rauschen
Verhältnis

(SNR) (SNR)

SNR maximal
für optimale
Rauschintensität