Prof. Dr. Tobias Brandes
Dr. Javier Cerrillo

10. Übungsblatt – TPVI: Quantensysteme im Nichtgleichgewicht

Abgabe: Mi. 01.02.2017 12:15 Uhr im Tutorium

Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Dreiergruppen erfolgen.

Aufgabe 16 (5 Punkte): *Unitarity of Scattering matrix*

Prove that the scattering matrix

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}$$

is unitary. Show that this implies the following relationships between the subblocks:

$$\begin{aligned} r^\dagger r + t^\dagger t = r'^\dagger r' + t'^\dagger t' &= \mathbb{1} \\ r r^\dagger + t' t'^\dagger = t t^\dagger + r' r'^\dagger &= \mathbb{1} \\ r^\dagger t' + t^\dagger r' = t'^\dagger r + r'^\dagger t &= 0 \\ r t^\dagger + t' r'^\dagger = t r^\dagger + r' t'^\dagger &= 0. \end{aligned}$$

Aufgabe 17 (8 Punkte): *Scatterers in series*

Consider two incoherent scatterers with transmission $probabilities$ T_1 and T_2.

- Show that the transmission probability of the two scatters in series is

$$T_{12} = \frac{T_1 T_2}{1 - R_1 R_2}; \quad \text{with} \quad R_i = 1 - T_i.$$

- Using this result, or otherwise, show that the total transmission probability of N identical scatterers in series in

$$T(N) = \frac{T}{N(1-T)+T}.$$

- Writing $N = \nu L$ where L is the sample length and ν the linear density of scatters, show that the transmission probability of length-L conductor can be written

$$T(L) = \frac{L_0}{L + L_0},$$

and find and interpret L_0.

Bitte Rückseite beachten!⟶

Aufgabe 18 (7 Punkte): *Quantum Point Contact*
The transmission probability of the nth channel of QPC is approximately

$$T_n(E) = \theta(E - \epsilon_n),$$

with mode energy $\epsilon_n = n\hbar\omega_0$ from a harmonic confinement in the absence of magnetic field. Calculate, plot and discuss the conductance of the QPC at finite temperature.

A magnetic field is applied to the sample in the plane of the 2DEG such that the electrons acquire a Zeeman splitting but with orbital degrees of freedom unaffected. Again: calculate the conductance, plot and discuss.

Hint: use the following expression for the conductance :

$$G = \frac{e^2}{h} \sum_n \int dE \; T_n(E) \left(- \frac{\partial f(E-\mu)}{\partial E} \right).$$